102nd Meeting of the Southeastern Section of the Mathematical Association of America
Abstracts of Papers and Posters

March 9–11, 2023

Contents

1 Invited Speakers

DS.1 *Wave Hello to Mathematical Modeling*
Nicole Panza (Francis Marion University)
Mathematical modeling is an important area because it allows us to use mathematics in interesting and robust ways to address issues in the world that are too costly or unsafe to investigate in actuality. Modeling is used in biology to accomplish just that. Two or three ovarian follicle waves typically occur per menstrual cycle. Nonlinear differential equation models representing the hormonal regulation of the menstrual cycle for a two-wave and a three-wave cycle are presented. The model is used to explore phenomena such as early menopause and superfecundation. An overall introduction to modeling and differential equations and this specific model and its applications will be presented.

DS.2 *Secret Lives of Mathematicians*
Aziza Jefferson (National Security Agency)
Mathematics can be more than just a subject in school; it can be a career. The government is the number one single employer of mathematicians in the country. Many of those mathematicians end up at the National Security Agency, where they find careers in research, information assurance, and cryptanalysis. This talk will be an introduction to the roles of mathematicians at NSA, as well as basics of cryptography.

GS.1 *Think Positive, You Might Be a Square!*
Vicki Powers (Emory University)
If a real polynomial f in n variables can be written as a sum of squares of real polynomials, then clearly f must take only nonnegative values in R^n, since the square of a real number is always nonnegative. This simple but powerful fact and generalizations of it underlie a large body of theoretical and computational results concerning positive polynomials and sums of squares. A natural question, which goes back to work of David Hilbert in the late 19th century, is whether a polynomial that takes only nonnegative values in R^n can be written as a sum of squares. Hilbert showed that in general the answer is "no". In this talk we take a leisurely stroll through results on sums of squares and positive polynomials. We will discuss the history of this topic, look at some famous results, and discuss a few of the many applications.

GS.2a *Mathematics, Music Theory, and Mentoring*
Vicky Klima (Appalachian State University)
In this talk, I will highlight some connections between algebraic structures and microtonal music theory, which, like many of my professional interests, began with a question from an undergraduate student. Much of what I have learned about mentoring has been obtained experimentally, through listening to students, and working with them as we explore their ideas together. This talk will emphasize how those lessons may translate into today's classroom.

GS.2b *Building Students' Confidence in Introductory Mathematics Courses*

Toyin Alli (University of Georgia)

It can be challenging to engage introductory level students who have a fear of math or believe they can never learn mathematics. Building an inclusive and encouraging classroom environment is at the forefront of my teaching approach. Entry activities for immediate participation along with group work and solution presentations build my students' confidence and provide a framework for solving problems on their own.

GS.3 *Can we make grace the norm in our classrooms?*

Candice Price (Smith College)

For much of my life, I have been confused about the way that people perceive the relationship between students and instructors in the classroom, especially in mathematics. There is such an adversarial relationship that even sharing my career choice with strangers leads to groans and stories of trauma. I believe this is the impact of creating a classroom without grace. So, when we add grace the opposite should happen, right? During our time together, I hope to discuss with you the ways that I incorporate grace into my classroom... and why many people think it is radical. I invite everyone to attend and reflect on ways they can make grace the norm in their classrooms and spaces.

2 Special Session: Preparing Students for Mathematical Careers

CAR1.1 *My Experience Teaching a PIC Math Course*

Laura Steil (Mars Hill University)
lsteil@mhu.edu

In this talk I will discuss my experience teaching a PIC Math (Preparation for Industrial Careers in Mathematical Sciences). This will include a look at the support offered by the MAA for this program, tips on how to find problems to use in these courses, and the daily structure of the course. Additionally, I will discuss ideas for recruiting students for this type of course and highlight its learning outcomes.

CAR1.2 *Preparing Students for Careers Through a Community Team Project Course*

Becky Sanft (University of North Carolina Asheville)
bsanft@unca.edu

The National Association of Colleges and Employers outlines career readiness competencies that prepare students for the workforce. These competencies include communication, teamwork, critical thinking, professionalism, and leadership. UNC Asheville has offered the course Community Team Project as a means for students to demonstrate these career readiness competencies. This course began as part of PIC Math (Preparation for Industrial Careers in Mathematical Sciences), an MAA and SIAM program funded by NSF that aims to equip students with crucial skills for a STEM career. A PIC Math class provides students with an authentic experience solving a real-world problem for business, industry, or government. I will provide an overview of PIC Math, logistics of the course, key factors that make this a positive experience for both the students and the sponsoring organizations, and challenges.

CAR1.3 *PIC Math - Community Engaged Scholarship*

Bree Ettinger (Emory University)
betting@emory.edu

In this talk I will share my experience teaching a spring semester research course developed through the MAA's PIC Math Program. In this course, teams of students worked on projects proposed by the Atlanta Beltline Inc, the group that oversees all aspects of the urban redevelopment project known as the Atlanta BeltLine, and Fair Fight Action, a nonprofit focused on combating voter suppression. For the Atlanta Beltline Inc, students used spatial data to identify safe walking and biking routes to Washington High School that utilize the BeltLine's existing Westside Trail. For Fair Fight Action, students analyzed the Georgia voter maintenance rolls and absentee ballot rejections. This talk will present details of the selected projects, the students' results, the techniques used, and the benefits and challenges of teaching this type of open-ended research course.

CAR1.4 *Infusing BIG preparation from lower-level classes to upper-level classes*

Amy Langville (College of Charleston)
langvillea@cofc.edu
Additional author(s):Kathryn Pedings-Behling, College of Charleston

In this talk, we will discuss several techniques that we have used at the College of Charleston to prepare our students for BIG (business, industry, and government) jobs. These techniques range from simple and subtle, such as well-chosen short activities or assignments with an additional communication component, to more intense and immersive experiences, such as group projects with "low-floor, high-ceiling" challenges and semester-long industrial projects. We will share lessons learned implementing these techniques in lower level courses such as calculus and upper level courses such as Optimization and Operations Research.

CAR1.5 *Working with Students through the PIC Math Program*
Laurie Zack (High Point University)
lzack@highpoint.edu
The benefits of gaining real world training cannot be replicated in the classroom and the PIC Math program is a great initial step to getting students connected with industry experience. In this talk, I will outline the PIC program and my personal achievements and struggles working with companies and with students solving such problems. I will also talk about ways you can adapt the program at your institution and focus on steps you can take to create a successful experience for both you and your students.

CAR2.1 *From Anywhere to Here: Secondary teaching opportunities for the next school year*
Ryan Fox (Belmont University)
ryan.fox@belmont.edu
Tradition suggests secondary school teachers need to complete their training while working on their content-area degrees. As demand for quality teachers increase while the number of traditionally-prepared teachers decreases, colleges and universities are adapting teacher preparation programs to meet these important needs. This presentation will focus on one such program while addressing why mathematics majors can become good teachers with targeted on-the-job training.

CAR2.2 *Teaching Opportunities for Undergraduate Mathematics Students*
Blake Dunshee (Belmont University)
blake.dunshee@belmont.edu
The saying "the best way to learn is to teach" was the inspiration for a recent collaboration between Belmont University and Vanderbilt Programs for Talented Youth. Seven Belmont math majors assisted with the design and implementation of a one day discrete math course for talented eighth and ninth grade students. This talk will focus on their experience and how it informs the way we teach discrete math to undergraduate students. We will discuss the benefits and challenges of having students peak behind the curtain into the world of teaching.

CAR2.3 *XPLoring Career Opportunities*
Laurie Heyer (Davidson College)
laheyer@davidson.edu
Experiential learning helps students prepare for a wide variety of possible careers. This talk describes three new models for experiential learning at Davidson College: (1) a micro-internship program that lets students explore more potential career opportunities than the typical semester- or summer-long internship, (2) a sponsored project program that connects external partners with faculty and students in a course, and (3) an experiential learning course, XPL 199, that helps students make the most of an internship by wrapping it in a curriculum that includes organizational culture, ethics and critical reflection.

CAR2.4 *Guiding Students to Data-Driven Solutions for a Local Nonprofit through a PIC Math Course*

Amanda Mangum (Converse University)
amanda.mangum@converse.edu

In Spring 2022, Converse University partnered with the Upstate Family Resource Center (UFRC), a local nonprofit that takes a comprehensive approach to providing assistance to families in need in Spartanburg County, South Carolina and the surrounding area. During the semester-long PIC Math course, the UFRC allowed the class to access data from assistance given in 2020 and 2021 through their main outreach, Harvest Family Ministries. This talk will give an overview of the data science projects that came out of this course and applicable skills students learned while also providing the necessary context of the history and mission of the UFRC.

CAR2.5 *Talk Back*

Caroline Maher-Boulis, Ph.D. (Lee University)
cmaherboulis@leeuniversity.edu

Please join us for discussion and Q&A with speakers from the session, including perspective and information from BIG SIGMAA chair Caroline Maher-Boulis. Share your own experiences and ideas, ask further questions of speakers, and discuss other ways to prepare students for mathematical careers.

3 Special Session: History and Philosophy of Mathematics

HPH.1 *Classical Constructions and Quadrature in Euclidean Geometry*

Dr. Ryan Thomas (Charleston Southern University)
rthomas@csuniv.edu
Additional author(s):Maddy Parsons, CSU undergraduate

Compass & straightedge constructions of plane figures formed a major component of the earliest stages of formal mathematics. For thousands of years, mathematicians used these basic tools to move the discipline forward. However, many of us have limited experience with geometric constructions. In this interactive talk we will examine some of the methods and major results firsthand, and look closely at one of the three classical problems handed down from antiquity: squaring the circle.

HPH.2 *Verrocchio's Palla and Spherical Geometry*

Doug Daniel (Presbyterian College)
ddaniel@presby.edu

In the 1460s and 1470s the sculptor and painter Verrocchio won the commission to create the "golden ball", called a palla, to sit atop Brunelleschi's masterpiece in Florence. It seems to have been created out of 8 copper plates with the help of the known spherical geometry of the time. How did he do it? This seems to be a question that has not been answered fully. Verrocchio's student, Leonardo, famously recorded drawings of some of the machines and soldering devises used to make the palla and place it in its perch. This story makes for a great jumping off point for spherical geometry, the theorem of Menelaos, and some of the historical figures working in Florence at the time, who used geometry to create great beauty.

HPH.3 *Approximating the Master: Reconciling Euler's Power Rule Proof with Modern Infinitesimal Analysis*

C. Bryan Dawson (Union University)
bdawson@uu.edu

A recent development in infinitesimal (nonstandard) analysis is the introduction of the approximation relation, which is utilized in the single-variable calculus textbook Calculus Set Free: Infinitesimals to the Rescue to simplify the calculation of limits (among other things). However, many of the resulting techniques are not new, but closely resemble those of the early masters. An example is Euler's proof of the power rule. In this presentation we use the modern infinitesimal methods to successfully analyze and justify Euler's proof, yielding insight that previously known methods are unable to provide.

HPH.4 *What is Mathematics: a la G. H. Hardy and Friends?*

Andrew Simoson (King University)
ajsimoso@king.edu

To give focus to a several minute talk on the nature of mathematics, we consider G.H. Hardy's 1940 classic defense of pure mathematics A Mathematician's Apology. As one approach to understanding his perspective, we leap briefly through history from Thales to Plato to Archimedes to Boethius and to, yes, Albrecht Durer and Alfred Nobel. We speculate on mathematics as the study of number and space, and its knack for spawning a myriad of related disciplines through the ages, and wonder about today's mathematical boundaries , and whether such a query is meaningful.

HPH.5 *Al-Khawarizmi Project through the MAA/HOM, A Call for Inclusivity*
AbdelNaser Al-Hasan (Newberry College)
naser.alhasan@newberry.edu
Additional author(s):None

Much of the research on the history of mathematics during the golden age of the Islamic empire (8th-16th century) in the past several decades has shown that most fundamental concepts of modern math and science are a legacy of the medieval Islamic Civilization. However, many in the West believe that the time from the end of the Greek era to the Renaissance was primarily a scientific desert. Such lack of inclusivity is due to an inaccurate narrative of history of science that has been in circulation for centuries. There is an imperative need to disseminate accurate information to the public.

4 Special Session: Modeling with Undergraduates

MOD1.1 *Modeling Change in a Preparation for Calculus Course*
Rachel Grotheer (Wofford College)
grotheerre@wofford.edu

In this talk, I will discuss how modeling takes center stage in a newly developed introductory course developed to prepare students for Calculus or mathematics-heavy STEM courses, called “Functions Modeling Change”. In the course, students gain understanding of the major function families not just through algebra and graphing, but also by exploring their behavior while using them to model data from various applications. The talk will specifically focus on the major modeling projects the students complete at the end of every unit, with topics such as modeling the relationship between human and pet age, and predicting sunrise and sunset times. I will explain how these same projects are extended and repeated in Calculus I class to help with transfer of knowledge, and discuss how exploring functions through modeling builds intuition that helps students in calculus and in applying those concepts in different contexts, such as science or economics courses.

MOD1.2 *Understanding Mathematical Models*

Rodica Cazacu (GCSU)
rodica.cazacu@gcsu.edu

My Introduction to Mathematical Modeling course is offered to mostly freshmen who are not math or science majors but must take a mathematics course in the general education area of their core. In this course students learn mostly about how to approach real life problems using the mathematics they already learned in their high school courses; therefore, they first must learn that one needs to understand the problem before trying to solve it. Finding the right model is as important as understanding why it is the best one and when it will stop working. This presentation will go over the way I built and continuously transformed this course so that my students will not just memorize some techniques and methods but understand why, when and how they need to look for tools this course provides. I will also talk about how I use same techniques in my upper-level math classes to apply the new concepts for solving real life problems at a different level.

MOD1.3 *Developing an Undergraduate Math Modeling Course: Perspectives from a Professor and Student*

Anastasia Wilson (Augusta University)
anawilson@augusta.edu
Additional author(s):Kayli Hendricks*, Augusta University *Denotes an undergraduate speaker

The Department of Mathematics at Augusta University is developing more of an applied math and math modeling emphasis in its undergraduate program. Additionally, the department recently started a Master of Science in Biological and Computational Mathematics degree for which a math modeling course was designed. Yet the department does not currently have an undergraduate math modeling course in the curriculum, nor is there much of an emphasis on math modeling in the lower-level math major courses such as Calculus. Feedback from the first cohort of graduate students taking the graduate math modeling course has prompted one of the presenters, Anastasia Wilson, to develop an undergraduate mathematics course, starting with offering it as a special topics course in Spring 2022. In this presentation, we will discuss our initial efforts to offer a math modeling course for undergraduates as a special topics course. One of the presenters, Kayli Hendricks, will talk about her experiences as a student who took the special topics course. We will also discuss changes made to the course design for the departmental proposal to add a math modeling course to the curriculum.

MOD1.4 *Designing and Running an Undergraduate Research Program in Mathematical Modeling*

Zach Abernathy (Winthrop University)
abernathyz@winthrop.edu

In this talk, we will share our experiences from running an undergraduate research program in mathematical modeling for the past 10+ years. These experiences have ranged from working with students on honors theses through hosting an NSF REU site. We will discuss our strategies for finding problems at an appropriate level, characteristics of a good model to explore, and our timeline for organizing a research project in modeling. We'll highlight some of the benefits and shortcomings of our approach with the aim of assisting anyone interested in working with students on modeling projects.

MOD2.1 *Research Experience for Undergraduates (REU) mentoring success story at Ursinus College*

Eric Takyi (Ursinus College)
etakyi@ursinus.edu

In this presentation, I will talk about my experience in mentoring a group of three undergraduates in formulating mathematical models which describe a food chain population dynamic. I will also discuss the goals (mathematical and personal growth) for the project and the approaches used in successfully publishing a research paper at the end of our summer Research Experience for Undergraduates (REU) at Ursinus College.

MOD2.2 *Modeling the Effects of Media on COVID-19 Transmission*

Eric Numfor (Augusta University)
enumfor@augusta.edu

The severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) that emerged in December 2019 is a zoonotic disease, derived from viral particles in bats. In this project, we formulate an SEIR model of SARS-CoV-2 with two susceptible classes comprising individuals who are unconscious to COVID-19 and those who are conscious to the virus due to media coverage. The disease-free equilibrium of our model is derived, and the basic reproduction number is computed. We studied the elasticity indices of the reproduction number with respect to each parameter and identified parameters that are most sensitive in increasing the reproduction number and those that are most sensitive in decreasing the reproduction number. Numerical simulations suggest that as more unconscious susceptible humans transition to conscious susceptible humans, there is a decrease in disease prevalence and a delay in the peak time of maximum prevalence in the population. Furthermore, an increase in the messaging rate of COVID-related information by conscious susceptible humans results in a decrease in the basic reproduction. Results of numerical simulations and contour plots highlight the importance of media in the transmission of SARS-CoV-2 in the population.

MOD2.3 *Stochastic Modeling and Forecasting of Covid 19 Deaths: Analysis for the Fifty States in the United States*
Olusegun Michael Otunuga (Augusta University)
`ootunuga@augusta.edu`
In this work, stochastic models are developed to study and analyze the infection cases and aggregate death counts of COVID-19 reported by the United States Centers for Disease Control and Prevention (CDC) for the fifty states in the United. The models derived capture irregularities in the sample path of the number of cases and the death counts better than the corresponding deterministic counterparts. The probability distributions of the number of cases and the death counts are derived, analyzed, and used to calculate the daily expected counts for each state, and to estimate epidemiological parameters in the models. A formula for calculating the expected passage time when the count is slowing down is derived. Forecasts for our simulation results are estimated.

5 Contributed Paper Sessions

CP1.1 *Perspectives on Secondary and Postsecondary Math Education from Educators Between Boundaries*
Jessica Kingsley (University of Tennessee-Knoxville)
`jkingsl1@vols.utk.edu`
Additional author(s):Anne Ho, Charlotte Beckford- University of Tennessee Knoxville
Past research typically assumes that an instructor is a high school or college instructor, but not both. The mechanisms to obtain teaching credentials for each are also traditionally separate, but some instructors teach at both levels simultaneously or transition between them during their careers. To better understand them, our team surveyed instructors with experience in both high school and college math teaching. For our qualitative study, we asked questions centered around the Pedagogical Content Knowledge domains within Mathematical Knowledge for Teaching (Ball et al., 2008; Shulman, 1986). In this talk, I will discuss the survey, data collection, coding, and findings on teacher perceptions of their jobs that fall across institutional boundaries. Specifically, I will detail the findings related to these instructors' perceptions of time pressures and systemic barriers when comparing high school and college math education.

CP1.2 *Encouraging Students to Write Mathematics: LaTeX and Mathematica*
Jeff Clark (Elon University)
`clarkj@elon.edu`
At the upper level of a mathematics program there are higher expectations for clarity in written exposition. Our program relies on two tools with differing strengths and weaknesses: Mathematica and LaTeX. This talk will compare and contrast these two tools as means of producing documents and presentations.

CP1.3 *Landing Planes, Marble slides and the Unit Circle. Using Desmos activities in Undergraduate Pre-Calculus Class*

Kevin D. LoPresto (Francis Marion University)
klopresto@fmarion.edu

In this session I will present a couple of activities created in the Desmos activity builder that I utilized in my undergraduate Pre-Calculus course. The activities are fun and engaging for students and provide multiple opportunities for them to discover and reflect on their own learning. Topics include transformations of functions, characteristics of linear functions and trigonometry using the unit circle.

CP1.4 *Course Development using Open Educational Resources and Pedagogical Practices*

Laura Lembeck (Western Carolina University)
lslembeck@wcu.edu

As a faculty participant in the Fall 2022 Open Pedagogy Incubator training sponsored by the University of North Carolina UNC System Scalable University Math SUM Institute for Open Course Development, I have been charged to 1) assess student learning of a particular topic in my Fall 2022 Quantitative Reasoning/Literacy (C2) liberal studies course and then 2) develop and implement teaching practices in my Spring 2023 classes that involve Open Pedagogical learning activities using Open Educational resources, and 3) survey students to assess the impact of the new activities. It's been great fun! I'm enjoying the new teaching pedagogies I am using, and my students are enjoying the engaging activities! I'll present my findings, surprises, changes to my curriculum, and students' responses to the changes!

CP1.5 *Disrupting the Expectation of Grades in Math Education*

Brandon Samples (Georgia College & State University)
brandon.samples@gcsu.edu

Have you ever wanted to move away from traditional grading but have been reluctant to do so? During my years as a teacher, I have moved towards the conclusion that grades are essentially worthless. Extensive research on grading has shown that replacing traditional grading with alternative approaches leads to a more substantial emphasis on meaningful student learning and growth. Such research suggests that the quality of student work improves and the shift towards an ownership of learning on the part of the student is greatly enhanced. Over the last three years, I have begun shifting my courses to operate with an "ungrading" approach, starting with my math teacher education courses and later incorporating my other mathematics courses. I would like to share some preliminary findings from my case-study research involving pre- and post-surveys as well as post-course interviews with former students. This research allows us to understand exactly how the students benefited from this structure and the impact it had on their learning and growth. Overall, the purpose of my study is to contribute to mathematics education research about the potential benefits of operating with this non-traditional assessment structure. I'd love to share my work so that you may also join me in disrupting the expectation of grades in math education.

CP1.6 *Project-based programming in a mathematics course*

Joe Barrera (Converse University)
joseph.barrera@converse.edu

Modern mathematicians understand the importance of computer programming in many facets of mathematics: research, problem solving, industry, and even recreation. In fact, many math majors have a computer science requirement. While computer science courses allow for creative ways to solve computer science problems, I wanted to design a course where that creativity could dovetail with a mathematically rigorous concept. My goal was to develop a course where students could use programming to solve problems related to mathematics—in particular, my students were tasked with producing fractal images. In this talk I will discuss the methods I used to build a project-based course centered on mathematical problem solving. I will also discuss lessons learned and how those may inform future iterations of the course. And yes, there will be plenty of fractal images throughout.

CP2.1 *Triangular and hexagonal Tangles*

Douglas A. Torrance (Piedmont University)
dtorrance@piedmont.edu

A Tangle is a smooth closed plane curve constructed by gluing together "links", or arcs of a circle. The existing literature has focused on the case where these links are quarter circles, which has a nice correspondence to the square tiling of the plane. There are two other regular tilings of the plane, one using equilateral triangles and another using regular hexagons. These give rise to Tangles whose links are sixths and thirds of circles, respectively. We introduce these objects and prove some basic results about them.

CP2.2 *A New Two-Person Magic Trick*

Colm Mulcahy (Spelman College)
colm@spelman.edu

Two-person card tricks based on mathematical principles go back at least 70 years: random cards are presented to Aodh who arranges them on the table in such a way that Bea – having previously seen nothing – can arrive on the scene, survey what is visible, and identify a card whose face is not visible. Early in 2023, a popular newspaper presented an ingenious puzzle which naturally adapts to a very surprising magic trick with or without cards.

CP2.3 *A Trek through Old-Growth Mathematics and New: How I designed a beautiful analog computer for the apparent solar time of sunrise and sunset*

Damon Scott (Francis Marion University)
DScott@FMarion.edu

After examining some formulas that have been known for hundreds of years, and plotting an elaborate diagram on a computer with the latest modern software, and then seeing points of intersection appear to lie on an ellipse, and then running through a substantial special case to determine the ellipse's proportions, I designed an analog computer that tells the apparent solar time of sunrise and sunset as a function of the time of year. Afterwards, I proved, rather than assumed, that the curve was indeed an ellipse. In this talk, I'll briefly tell of these adventures and provide every member of the audience with his own, beautiful analog computer custom-designed for his own latitude to the nearest degree.

CP2.4 *How many cards should you lay out in the card game EvenQuads?*

Timothy E. Goldberg (Lenoir-Rhyne University)
timothy.goldberg@gmail.com
Additional author(s):Julia Crager, Bard College Felicia Flores, Bard College Lauren L. Rose, Bard College Daniel Rose-Levine, Bard College Darrion Thornburgh, Bard College Raphael Walker, Université Paris Saclay

EvenQuads is a card game published by the Association for Women in Mathematics based on the game SuperSET (later Quads) invented by Rose and Perreira in 2013. It is similar to the game SET, but with cards whose symbol attributes each have four possible values instead of three. Players lay out a certain number of cards and then look for special collections of four cards, called *quads*. As in SET, if there are no quads, then the players deal out another card. There are various ways to assign coordinates to the cards that makes the EvenQuads deck equivalent to a finite vector space, and a model for the finite affine geometry $AG(6, 2)$. Viewed this way, four cards form a quad if and only if they form a plane.

A question of great interest in finite geometry regards the possible sizes and structures of sets of points containing no lines. For EvenQuads, since any two points form a line, we instead study sets of points containing no planes, or equivalently sets of EvenQuads cards containing no quads. In this presentation, we shall present recent work and results regarding such sets in $AG(6, 2)$ and more generally in $AG(n, 2)$. As a consequence, we will compute the probability of finding a quad in a layout of k random EvenQuads cards. This will tell us how likely it is that players will have to deal an extra card depending on how many they laid out to start, thus answering the question posed in the title of this presentation.

This is based on joint work with Crager et al.

CP2.5 *HACKENBUSH*

Cannon McIntosh (University of Tennessee Knoxville)
cmcinto8@vols.utk.edu

A playful stroll into the world of combinatorial game theory. Hackenbush is a simple game to learn, but analyzing its strategy will fundamentally change the way you view numbers.

CP2.6 *Random Arrays of Toothpicks*

Tom Lewis (Furman University)
tom.lewis@furman.edu

The inspiration for this topic was the Numberphile video Terrific Toothpick Patterns presented by Neil Sloane. In this talk, we introduce a random model of toothpick evolution and discuss some possible avenues of investigation.

CP3.1 *Two Ways of Looking at Limits*

Barrett Walls (Georgia State University)
bwalls@gsu.edu
Additional author(s):Iason Rusodimos - Georgia State University

We present two very different ways of solving a difficult limit problem, which originally appeared in the MAA Monthly. One is by finding a closed form solution and the other is without one. We discuss ways these can be presented to calculus students to increase their understanding of the concept and methods for calculating limits.

CP3.2 *Existence and multiplicity results for classes of elliptic boundary value problems*

Nalin Fonseka (Carolina University)
fonsekan@carolinau.edu
Additional author(s):R. Shivaji (University of North Carolina Greensboro) Byungjae Son (Ohio Northern University) K. Spetzer (University of North Carolina Greensboro)

We study positive solutions to steady state reaction diffusion equations of the form:

$$\begin{cases} -\Delta u = \lambda f(u);\ \Omega, \\ \frac{\partial u}{\partial \eta} + \mu(\lambda)u = 0; \partial\Omega, \end{cases}$$

where $\lambda > 0$, Ω is a bounded domain in $\mathbb{R}^N$; $N \geq 1$ with smooth boundary $\partial\Omega$, $\frac{\partial u}{\partial \eta}$ is the outward normal derivative of u, $\mu \in C([0,\infty))$ is strictly increasing such that $\mu(0) \geq 0$ and $f \in C^2([0,r_0))$ with $0 < r_0 \leq \infty$. If $r_0 < \infty$ we assume $f \in C^2([0,r_0])$ with $f(r_0) = 0$ and $f(s) \leq 0$ for $s \in (r_0,\infty)$, and if $r_0 = \infty$ we assume $\lim_{s\to\infty} f(s) > 0$ and $\lim_{s\to\infty} \frac{f(s)}{s} = 0$ (sublinear at ∞). Note here that the parameter λ influences both the equation and the boundary condition. We discuss existence, nonexistence, multiplicity and uniqueness results for the cases when (A) $f(0) = 0$, (B) $f(0) < 0$, and (C) $f(0) > 0$. We obtain existence and multiplicity results by the method of sub-super solutions and uniqueness results by comparison principles and a priori estimates.

CP3.3 *Convexity and Inequalities*

Risto Atanasov (Western Carolina University)
ratanasov@wcu.edu

As convex functions are defined by a functional inequality, they lead to a number of important inequalities. They can be used in proving other inequalities, particularly those appearing as problems on mathematical competitions, including William Lowell Putnam Mathematical Competition.

CP3.4 *Properties and Artistic Qualities of the Iterates of Complex Poles and Critical Points*

Julie Barnes (Western Carolina University)
jbarnes@email.wcu.edu
Additional author(s):Beth Schaubroeck, United States Air Force Academy

We explore the family of rational functions $f(z) = \frac{(z-1)^k}{z^n}$, examining their critical points and poles under finite iteration. The contour diagrams of the real parts of these iterates reveal the behavior of the critical points and poles of each function in beautiful ways. The poles appear as "flowers" and the critical points appear as standard saddle shapes whose contours are intersecting curves or curves that bend near the critical point. We describe, for this family of functions, how orders of the critical points and poles in the original function affect the shapes of the contours after repeated composition of that function with itself (iteration). This in turn tells us information about properties of the critical points and poles under iteration. These properties also give us insight on how to create artwork from other complex functions based on their critical points and poles.

CP3.5 *Initiating Undergraduates to Mathematical Research: Case of the Simplified One-Dimensional Saturation Equation*

Koffi B Fadimba (University of South Carolina Aiken)
koffif@usca.edu

The goal is to have undergraduate students study the simplified one dimensional saturation equation $\frac{\partial S}{\partial} + (f(s))_x = (k(S)S_x)_x$ in the interval $[0, T] \times [0, 1]$. We start with the simple differential equation $-u'' = f$ on the interval $[0, 1]$, with boundary conditions at $x = 0$ and $x = 1$. Though we can find the exact solution of this equation, we have students approximate the solutions in two ways. The first way is the finite element method: multiplying both sides of the equation by a test function, integrating both sides on $[0, 1]$ and then using integration by parts (variational formulation followed by a finite element discretization). The second way is the finite difference method using numerical differentiation. This will introduce them to the two main methods used in numerical approximations of differential equations. The next step to make the equation more complicated in some way. For example by considering the partial differential equation $\frac{\partial S}{\partial t} = (S)_{xx}$ and use, for instance, a fully implicit difference method in time and space to approximate a solution. Do this gradually until we consider the full equation. In this talk we present some of these gradual steps in theory and in practice.

CP3.6 *Monotonicity results for fractional difference operators*

Raj Dahal (Coastal Carolina University)
rdahal@coastal.edu

We will introduce discrete fractional calculus from its basics, its developments over the years and their applications. We also show, mostly by means of numerical simulation, that it is possible to deduce information about the monotonicity of a function f in spite of the non-positivity of the discrete forward difference of the function f.

CP4.1 *A Systematic Method to Model Natural Variability and Uncertainty in Diffusive Transport*

Martin L. Tanaka (Western Carolina University)
mtanaka@wcu.edu
Additional author(s):John Wagaman - Western Carolina University, Cullowhee, NC USA; David M. Saylor - US Food and Drug Administration, Silver Spring, MD, USA

Implanted polymeric medical devices have the potential to leach hazardous materials into the body. Often the worst-case conditions are used to approximate the amount of material that could be released. However, this approach only yields a single scalar value. A more clinically relevant approach is to model the input parameters as probability density functions. Our approach includes dividing the parameter space into an n dimensional grid where n is the number of input parameters. Each combination of the grid was then evaluated systematically by calculating both the quantitative value of material released and the likelihood for each grid combination. These results were compared to results obtained using a Monte Carlo simulation. Analysis showed the systematic approach to yield densely represented data over the entire range of input parameter values while the Monte Carlo simulation becomes sparse at extreme values of the input parameters. Thus, this systematic approach may be advantageous when results over the entire range of input parameter space is desired.

CP4.2 *The Mathematics of Options Trading: How to become successful in the Stock Market*

Robert S. Owor (Albany State University)
robert.owor@asurams.edu

Broadly speaking, options trading refers to the practice of buying and selling options contracts. These contracts give the buyer the right – but not the obligation to buy or sell a stock or other asset at a predetermined price, within a predetermined time period. In this paper, we examine the mathematics of options trading and how one can use it to make profits in the stock market. It is widely acknowledged that there has been a major breakthrough in the mathematical theory of option trading. This breakthrough, which is usually summarized by the Black-Scholes formula, has generated a lot of enthusiasm and a certain respect for Mathematicians in the Stock Market. On the mathematical side, it involves advanced probabilistic and statistical techniques from martingale theory and stochastic calculus which are accessible only to a small group of experts with a high degree of mathematical prowess; thus the prestige. In its practical implications it offers exciting prospects. Its promise is that, by a suitable choice of a trading strategy, the risk involved in handling an option can be eliminated completely.

CP4.3 *Cost-Benefit Analysis of Two Retirement Products – ROTH IRA and IUL*

Jayanti Rani Saha (Albany State University)
jayanti.saha@asurams.edu
Additional author(s):Jasane M. Baker, Arun Kumar Saha, Albany State University

Every working person desires to get a comfortable retirement money at the end of service. For that reason, there are various money savings schemes such as 401(k), 403(b), 457(b), ROTH IRA and Traditional IRA which are very common to the public. Recently, another retirement savings tool called "Indexed Universal Life (IUL)" insurance product is gaining its momentum in the financial world. This is actually a life insurance product but is designed for retirement savings and tax free payment after retirement. In IUL, an employee can save unlimited amount of money whereas in ROTH IRA contribution is limited by IRS based on annual income. IUL, when designed properly, can provide income stream for whole life or 120 years maximum whereas fund in ROTH IRA can be dried up before death. Yearly premium for an IUL is high while cost of maintaining mutual funds in ROTH IRA is low. Therefore, in this project, a cost-benefit analysis of IUL and ROTH IRA has been performed.

CP4.4 *Modeling effects of matrix heterogeneity on population persistence at the patch-level*

Dustin Nichols (UNC Greensboro)
dtnichol@uncg.edu
Additional author(s):Nalin Fonseka (Carolina University), Jerome Goddard II (Auburn University at Montgomery) Alketa Henderson (UNC Greensboro), Ratnasingham Shivaji (UNC Greensboro)

Habitat loss and fragmentation is the largest contributing factor to species extinction and declining biodiversity. Landscapes are becoming highly spatially heterogeneous with varying degrees of human modification. Much theoretical study of habitat fragmentation has historically focused on a simple theoretical landscape with patches of habitat surrounded by a spatially homogeneous hostile matrix. However, terrestrial habitat patches are often surrounded by complex mosaics of many different land cover types, which are rarely ecologically neutral or completely inhospitable environments. We employ an extension of a reaction diffusion model to explore effects of heterogeneity in the matrix immediately surrounding a patch in a one-dimensional theoretical landscape. Exact dynamics of a population exhibiting logistic growth, an unbiased random walk in the patch and matrix, habitat preference at the patch/matrix interface, and two functionally different matrix types for the one-dimensional landscape is obtained. These results show existence of a minimum patch size (MPS), below which population persistence is not possible. This MPS can be estimated via empirically derived estimates of patch intrinsic growth rate and diffusion rate, habitat preference, and matrix death and diffusion rates. We conclude that local matrix heterogeneity can greatly change model predictions, and argue that conservation strategies should not only consider patch size, configuration, and quality, but also quality and spatial structure of the surrounding matrix.

CP4.5 *On the effects of density-dependent dispersal on ecological models with logistic and weak Allee type growth terms*

Alketa Henderson (University of North Carolina Greensboro)
ahhende2@uncg.edu
Additional author(s):Ananta Acharya(University of North Carolina Greensboro) Nalin Fonseka (Carolina University) Jerome Goddard II (Auburn University Montgomery) R. Shivaji (University of North Carolina Greensboro)

We analyse positive solutions to the steady states reaction diffusion equation:

$$\begin{cases} -\Delta u = \lambda \frac{1}{a} u(1-u)(a+u); \ \Omega \\ \frac{\partial u}{\partial \eta} + \gamma \sqrt{\lambda} g(u) u = 0; \ \partial\Omega \end{cases}$$

where $a > 0, \gamma > 0$, and $\lambda > 0$ are parameters, Ω is a bounded domain in $\mathbb{R}^N$; $N > 1$ with smooth boundary $\partial\Omega$ or $\Omega = (0,1)$, $\frac{\partial u}{\partial \eta}$ is the outward normal derivative of u. In this paper, we study three emigration forms. Namely, we consider density independent emigration ($g = 1$), a negative density dependent emigration of the form $g(s) = \frac{1}{1+\beta s}$, and a positive density dependent emigration of the form $g(s) = 1 + \beta s$, where $\beta > 0$ is a parameter. We establish existence, nonexistence, and multiplicity results for ranges of λ depending on the choice of the function g. We consider the case when $a \geq 1$ (logistic type growth) and the case when $0 < a < 1$ (weak Allee effect type growth). We prove our existence and multiplicity results via the method of sub-super-solutions. When $\Omega = (0,1)$, we also provide the exact bifurcation diagrams for positive solutions for certain values of the parameters a, β and γ via a quadrature method and mathematica computations.

CP5.1 *A Remark on Linear-Quadratic Problems in Sequence Spaces*

Zephyrinus Okonkwo (Albany State University)
Zephyrinus.Okonkwo@asurams.edu
Additional author(s):Anil Devarapu, Department of Mathematics, University of North Georgia, Gainesville, GA

In this talk, we consider systems of the form $x(k+1) = A(k)x(k) + B(k)u(k), K \geq 0$, where the trajectories $x(k) \in R^n$ and the controls $u(k) \in R^m, m < n.A(k), K \geq 0$ $n \times n$ invertible matrices while $B(k)$ is a sequence of $n \times m$ matrices whose entry are in $l^\infty(Z_+, R)$, $Z_+ = \{1, 2, 3, ...\}$. Subsequently, we discuss the existence of the solutions of the above equation as well the associated optimal control problem

CP5.2 *Properties of Ulam Sequences*

Breeanne Baker Swart (The Citadel)
bbaker2@citadel.edu
Additional author(s):Antara Mukherjee, The Citadel; Aaron Reynolds, The Citadel, undergraduate student

For positive integers $u < v$, the (u, v)-Ulam sequence, $(u, v) = s_1, s_2, s_3, \ldots$, is defined by $s_1 = u$, $s_2 = v$, and s_i, $i \geq 3$, is the least integer greater than s_{i-1} having a unique representation in the form $s_i = s_j + s_k$ where $1 \leq j < k \leq i-1$. We will focus on the $(1, n)$-Ulam sequences, $n \geq 1$, and discuss properties of subsequences of consecutive integers.

CP5.3 *Application of Shooting Method to Unsteady Mixed Convection Problem*
Anil Devarapu (University of North Georgia)
adevarapu@ung.edu
Additional author(s):Zephyrinus Okonkwo

The study of mixed convection heat transfer has garnered significant interest across various industries due to its widespread practical uses. In this talk, we discuss the implementation of shooting method to unsteady mixed convection boundary layer equations. The mixed convection problem is governed by a system of nonlinear differential equations, which are solved numerically using a combination of the shooting method and Runge Kutta method. Numerical results are presented for the skin friction coefficient, the local Nusselt number and the local Sherwood number as well as for the velocity, temperature profiles.

CP5.4 *Mathematical model of erosion and deposition in porous media with branching structure.*
Emeka Peter Mazi (Georgia State University)
emazi1@student.gsu.edu
Additional author(s):Hamad El Kahza Department of Mathematical Sciences, University of Delaware Prof. Pejman Sanaei Department of Mathematics and Statistics, Georgia State University

Erosion and deposition can have significant effects in the nurture, industrial applications and in general porous media. In this work, we study the deposition and erosion of solid particles at a microscale level and their direct consequence on the internal structure of porous media with complex internal morphology. We present an idealized model, in which a porous medium consists of bifurcating cylindrical channels. The flow and solid particles are modeled by Stokes and advection-diffusion equations, respectively. Finally, we investigate the erosion and deposition of solid particles and characterize the evolution of the internal geometry of porous media.

CP5.5 *How frequent are voting anomalies in US Ranked-Choice elections?*
Adam Graham-Squire (High Point University)
agrahams@highpoint.edu
Additional author(s):David McCune, William Jewel College

Instant-Runoff Voting (IRV) is known to have many flaws. IRV can fail to elect a Condorcet winner and is susceptible to monotonicity paradoxes and the spoiler effect, for example. We use a database of 182 American ranked-choice elections for political office from the years 2004-2022 to investigate empirically how frequently IRV's deficiencies manifest in practice. Our general finding is that IRV's weaknesses are rarely observed in real-world elections, with the exception that ballot exhaustion frequently causes majoritarian failures. Instructors who teach courses involving Voting Theory will find interesting real-world examples in this talk that you can share with your classes.

CP6.1 *Math Hands – Geometry*

L. Jeneva Clark (University of Tennessee, Knoxville)
dr.jenevaclark@utk.edu
Additional author(s):Philip Rosenbalm (an undergraduate student from the University of Tennessee)

Do math with your hands! Many are familiar with Piaget's philosophy (1976) that children learn with hands-on approaches; however, the Piagetian proverb need not preclude adults from also rolling up their sleeves and engaging in hands-on mathematical play (Carbonneau, Marley, & Selig, 2012). In fact, embodied cognition (Wilson, 2002) suggests that our motor system influences our thinking, instead of solely vice versa.

In an undergraduate geometry course for math majors at the University of Tennessee, hands-on approaches are used to reason through common proofs and theorems in Euclidean geometry. For example, students use physics lab equipment to verify Newton's second law and then prove the centroid theorem. Students become familiar with the parallel postulate using hyperbolic paper planes made from packing tape. Students act out proofs using ropes, paper folding, and pancakes.

When hands-on activities were embedded in a math course at the University of Tennessee, 85% of the students reported, "Hands-on activities help me because they allow me to see things differently," 73% reported, "Hands-on activities help me because they make it easier to discuss concepts with others," and 72% reported, "Hands-on activities help me because they allow me to touch and move things." Come give it a try yourself, and bring your math hands!

CP6.2 *Demystifying Logarithms by Introducing Log Notation Before Log Concepts*

Kelly Buch (Austin Peay State University)
buchk@apsu.edu

Many students struggle to understand logarithms. Logarithms are indeed conceptually difficult for these students because they are asked to think in the opposite direction of what they are used to. However, the notation used for logarithms can be an additional barrier to learning. Logarithms are written using notation most students have never seen before. An abbreviated word, instead of familiar symbols, represents the rule. So, while students are learning to think in a new way, students are also learning to read and write mathematics in a new way. Perhaps by introducing students to the notation of logarithms before the concept of logarithms, students won't be hindered by the notation. In this talk, I'll share an activity I created and implemented with precalculus students that was designed to introduce the style of notation used with logarithms using rules they are already familiar with-addition and subtraction. By demystifying this new notation before introducing the concept of logarithms, students were prepared to tackle the conceptual difficulties of logarithms with a now-familiar notation. While this work is not part of formalized educational research, I hope to share a perspective of interest to other educators of precalculus students.

CP6.3 *Too Much of a Good Thing: How to Navigate the Overabundance of Online Resources*

Denise Dawson (Charleston Southern University)
ethomas@csuniv.edu
Additional author(s):Emily Thomas, Charleston Southern University

Today's students are seeking help from the internet more than ever, and this is changing our experience in the world of teaching Math. We know how powerful and helpful it can be to have the right resources readily available. As teachers we want to be the first line of "defense" for our students' understanding; however, we are not available at all hours when they are studying and working. Students are turning to the hundreds of videos on each topic and the apps where you simply take a picture of the problem and get a full solution. They are using these apps and websites with blind trust. We will share our experiences with this at Charleston Southern University, including anecdotes from the classroom, some of the strategies we've used to address the issue, and a few of the setbacks we've encountered.

CP6.4 *Math Teachers' Circle's: Connect with your Community*

Brandon Rupinski (Western Carolina University)
brupinski@wcu.edu

In this talk we will define a Math Teachers' Circle, discuss community benefits, share digital resources and engage in a short demonstration.

CP6.5 *Some Positive Impacts of the Study Table Project at Albany State University*

Zephyrinus Okonkwo (Albany State University)
Zephyrinus.Okonkwo@asurams.edu
Additional author(s):Robert S. Owor

Many students come to Albany State University with a variety of preparations and mindsets. For the freshman cohort, their first year at ASU is mostly challenging due to change in environment, independence, higher expectations, and their being held accountable for their success. By providing to them a set of student success activities, many of them who avail themselves of these opportunities, are able to progress adequately in their courses and programs. The goal of the Study Table is Student Success. Study Table activities has impact on student learning, student achievement and persistence at Albany State University. This could subsequently result in the improvement of student retention, progression, and degree attainment, ultimately have broader impact. In this talk, we will examine the Study program and its impact at Albany State University.

CP6.6 *Addressing the Challenges of Using Standards-Based Grading in Precalculus*
Rachel Epstein (Georgia College)
rachel.epstein@gcsu.edu
Standards-based grading (SBG), also known as mastery grading, gives students multiple opportunities to demonstrate proficiency in the course standards or objectives. Instead of earning a percentage on each test or assignment, work is scored on whether the standard has been met, then students are given the opportunity to reassess. It is designed to reduce student testing anxiety while increasing the depth of understanding. In this presentation, I will discuss the results of a study where I implemented SBG in a college Precalculus course during the Covid-19 pandemic and surveyed the students about their experiences. Students shared how their learning has been impacted by the pandemic and their thoughts on taking a course that used SBG during the pandemic. While there were many benefits to using SBG, it also comes with unique challenges. I will discuss these challenges and how I am attempting to address them this semester.

CP7.1 *Simple Solution to an Old Problem (Graph Theory)*
Anil K. Saxena (Coastal Carolina University)
aksaxena@coastal.edu
Additional author(s):Subhash C. Saxena, Distinguished Professor Emeritus
Gardener's Problem: how to plant 19 trees in such a way that there are exactly 9 rows, with each row containing exactly 5 trees. I will talk about the history of this problem as well as present a solution to the problem.

CP7.2 *Examining clusters in chromatic and Tutte polynomial data*
Dan Scofield (Francis Marion University)
daniel.scofield@fmarion.edu
Additional author(s):Radmila Sazdanovic, North Carolina State University
We apply methods from topological data analysis to examine features of high-dimensional data sets that encode the chromatic and Tutte polynomials of graphs up to 10 vertices. We provide evidence for a relationship between the coefficients of the chromatic polynomial and graph irregularity, and relate our insights to existing results about threshold graphs and bipartite graphs. These methods are further extended to illuminate the structure of data collected from the Tutte polynomial.

CP7.3 *A Note on the Largest Clique in a Graph*

Earl Barnes (Georgia Tech)
earl1942barnes@yahoo.com

Let G be a graph on n nodes $\{1, 2, \ldots, n\}$. Let $\chi(G)$ denote the minimum number of colors needed to color the nodes of G so that adjacent nodes get different colors. Let $\omega(G)$ denote the size of the largest clique in G and let $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$ denote the eigenvalues of the adjacency matrix for G. There is no efficient algorithm for computing $\omega(G)$ or $\chi(G)$, But certain bounds for these quantities are known. For example it is known that

$$1 + \frac{\lambda_1}{|\lambda_n|} \leq \chi(G) \leq 1 + \lambda_1 \text{ and clearly } \omega(G) \leq \chi(G.$$

We will show that, with some mild assumptions

$$\omega(G) \leq \frac{\lambda_1 + \lambda_n + \sqrt{(\lambda_1 - \lambda_n)^2 - 4d}}{2} + 1, \text{ where } d = \frac{n}{\lambda_1 + 1} - 1.$$

Equality holds if G is a clique.

CP7.4 *Mathematical challenges - first impressions*

George Cazacu (Georgia College)
george.cazacu@gcsu.edu

A collection of examples from various fields of mathematics are presented to illustrate the delicate role of intuition when tackling mathematical questions. This talk is addressed mainly to undergraduate researchers, but it can be of general interest.

CP7.5 *From CMJ to CMC: Group of Four KenKen*

Stephen Davis (Davidson College)
stdavis@davidson.edu

The Charlotte Math Club provides enrichment and challenges for middle and high school students who have a strong interest and ability in mathematics. During our monthly meetings, we include an exploration of a mathematical topic not usually encountered in the school curriculum. Recently, I've found inspiration for topics in the College Mathematics Journal. This talk presents an overview of one such inspiration, investigating KenKen puzzles on a 4×4 grid populated either by elements from $\mathbb{Z}_4$ or by elements from the Klein 4-group. (See David Nacin's 2017 and 2022 articles, CMJ 48:4 and 53:4.) Along the way we get to do some counting and lots of play that reveals and reinforces the structure and properties of these two groups.

CP7.6 *A comparative study of elementary mathematics textbooks from three countries based on the topic of fractions*

Deepak Basyal (Coastal Carolina University)
dbasyal@coastal.edu

Studies show that inability to learn fractions properly in elementary schools inhibits students' ability to learn further mathematical concepts. Instructional materials play an important role in teaching and learning of fractions. Thus, comparative investigations of instructional materials such as textbooks can provide valuable insights that can be useful in effective teaching and learning of fractions. This study compares the topic of fractions in elementary textbook series from the USA, Finland, and Nepal. Our sample consisted of the Everyday Mathematics series published by McGraw Hill (USA), Laskutaito Mathematics published by Finland Werner Söderström Corporation (Finland), and My Mathematics series published by Curriculum Development Center of Government of Nepal. The preliminary results suggest that the students in these countries have varying opportunities to learn fractions. Implications of the study regarding teaching and learning of fraction concepts will be discussed.

CP8.1 *Circulant Quaternionic Hadamard matrices of order 5*

Chase Worley (Lander University)
cworley@lander.edu
Additional author(s):Logan Higginbotham, Campbell University

We begin by recalling classification results of real and complex Hadamard matrices of small order. We then proceed to the quaternionic cases of small orders in particular up to order 5. We then find a one-parameter family of circulant quaternionic Hadamard matrices of order 5. We find that for even small orders the quaternion case is much richer than the complex case. This is joint work with Logan Higginbotham.

CP8.2 *The Ax-Grothendieck Theorem and its Strange Proof*

Alex Foster (University of North Carolina at Chapel Hill)
aofoster@live.unc.edu

The Ax-Grothendieck theorem says that any injective polynomial function from $\mathbb{C}^n$ to $\mathbb{C}^n$ is surjective. If $\mathbb{C}$ was a finite set, this would be trivial, as any injective function from a finite set to itself is surjective. In no reasonable world would we be convinced by this alone, but a real proof of the Ax-Grothendieck theorem starts here. We will discuss this theorem first for finite fields, then for their algebraic closures, and finally, using a strange fact from the world of model theory, we will conclude the theorem for $\mathbb{C}$.

CP8.3 *Thin Groups*

Cannon McIntosh (University of Tennessee Knoxville)
cmcinto8@vols.utk.edu

A thin group is a finitely generated infinite index subgroup of an arithmetic group that is Zariski dense in the corresponding algebraic group. Our goal is to acquaint ourselves with thin groups by analyzing nonexamples and examples.

CP8.4 *Exact zero-divisor subgraphs*

Justin Hoffmeier (Florida Polytechnic University)
jhoffmeier@floridapoly.edu

In algebra we often use the zero product property that states two non-zero complex numbers cannot have a product that is zero. However, there are many mathematical systems where this statement fails: if the product of two non-zero elements is zero then they are called zero-divisors. For a zero-divisor x, the annihilator of x is the set of all elements whose product with x is zero. In some cases, the annihilators are generated by just one element and can exhibit a certain duality which leads to the notion of an "exact" zero-divisor. In this talk we will identify these types of annihilators from a graph of zero-divisors. By focusing on the integers modulo n, this talk is very accessible for those of all backgrounds.

CP8.5 *Set-Valued Dynamical Systems and thier Inverse Limits*

Tavish Dunn (Emory University Oxford College)
tdunn5@emory.edu
Additional author(s):David Ryden

Inverse limits of a single function provide a bridge between topology and dynamical systems. In this talk, I will discuss the inverse limits of set-valued functions $F : [0,1] \to 2^{[0,1]}$, exploring how dynamical properties of F inform the topology of the inverse limit and vice-versa.

CP8.6 *Some identities in generalized arithmetic triangles*

Doug Ensley (Shippensburg University)
dougensley@gmail.com
Additional author(s):Jesica Hoover (student) and Ji Young Choi, both of Shippensburg University

A natural generalization of Pascal's Triangle comes from relaxing the initial condition that the numbers down the sides are all 1s. Within this context we generalize some common Pascal-triangle identities using tools from beginning combinatorics. In this presentation, we will share results and the framework in which we arrived at them, opening up the door for additional research efforts by anyone interested to play along in this very accessible sandbox. This work is related to an undergraduate research project that took place during the summer of 2022.

6 Undergraduate Paper Sessions

UT1.1 *The Good and Bad of Sequences I*
Ernest James (The Citadel)
ejames3@citadel.edu
We solved problem 2146 from Math Magazine. The problem consisted of four parts dealing with the arithmetic sequence

$$\sum_{i=0}^{k-1}(a+di) = \sum_{i=k}^{n-1}(a+di).$$

We identified if certain values for a and d resulted in a good or bad case. A bad case was defined as a case where the arithmetic sequences did not exist and a good case was defined as a case where a sequence did exist. In this presentation we will prove two cases where a and d will result in a bad case.

UT1.2 *The Good and Bad of Sequences II*
William Boyd (The Citadel)
wboyd4@citadel.edu
Additional author(s):Ernest James-Coauthor
We solved problem 2146 from Math Magazine. The problem consisted of four parts dealing with the arithmetic sequence

$$\sum_{i=0}^{k-1}(a+di) = \sum_{i=k}^{n-1}(a+di).$$

We identified if certain values for a and d resulted in a good or bad case. A bad case was defined as a case where the arithmetic sequences did not exist and a good case was defined as a case where a sequence did exist. In this presentation, we will demonstrate two cases where for some particular values of a and d we have a good case.

UT1.3 *An analysis of the sequence x(n+2) = imx(n+1) + x(n)*
Jensen Meade (Coastal Carolina University)
jrmeade@coastal.edu
Additional author(s):Prashant Sansigry, Ogul Duncan, David Duncan, Coastal Carolina University
In this presentation we analyze the sequence x(n+2) = imx(n+1) + x(n), with x(1) = x(2) = 1 + I, where I is imaginary and m is real. Plotting the sequence for different values of m, we see interesting figures from the conic sections. For values of m in the interval (-2,2), we prove that the figures generated are ellipses. We also provide analysis which prove that for certain values of m, the sequence generated is periodic with even period.

UT1.4 *Finite Difference Scheme for the one-dimensional saturation equation*

Artis Carter (Augusta University)
artcarter@augusta.edu
Additional author(s):Jonathan Acuna-Robles (Augusta University), Alexandria Carter (Augusta University), Dr. He Yang (Augusta University)

In this talk, I will introduce a finite difference method for the one-dimensional saturation equation. The saturation equation is a convection-diffusion equation which is derived from the modeling of a two-phase immiscible flow through a porous medium. I will discuss the easy implementation of the finite difference method and present its properties when solving the saturation equation. Several numerical examples will also be given to demonstrate the performance of the method.

UT1.5 *Homotopy Analysis Method for the One-Dimensional Saturation Equation*

Jonathan Acuna Robles (Augusta Univeristy)
jacunarobles@augusta.edu
Additional author(s):Alexandria Carter: Augusta University Artis Carter: Augusta University He Yang: Augusta University

In this presentation, we consider an analytical approximation method to solve the saturation equation. The equation is a model that describes a flow in which 2 different substances are simultaneously present, and these 2 substances do not mix to give a single phase, such as oil and water, or a mudflow. In addition, the flow moves in a material that contains sufficient open space for the flow to pass through. The model has important applications in the oil industry. I will discuss how the Homotopy Analysis Method is used to solve this problem. The Homotopy Analysis Method is a generalized homotopy perturbation method with the addition of a control parameter for convergence. I will use severaal numerical examples to demonstrate the influence of such control parameter to the accuracy of the approximation solutions.

UT2.1 *Utilizing Entropy and Volatility for Machine Aided Classification of Atrial Fibrillation*

Arnav Gupta (Ravenscroft School)
arnavg363@gmail.com
Additional author(s):Luke Ni, Chapel Hill High School, High School Student

Atrial Fibrillation (A-FIB) is the most common heartbeat arrhythmia in the United States. It is characterized by an irregularity in electrical impulse reception, causing the atria to beat asynchronously from the ventricles. A-FIB episodes are sporadic, so symptoms may not appear during clinical visits, causing a risk of misdiagnosis which can lead to life-threatening strokes. This study seeks to utilize machine-aided analysis to detect A-FIB through the entropy and volatility of RR-intervals, the distance between two R-peaks, in electrocardiograms (ECG). Data was collected from the Massachusetts Institute of Technology and Beth-Israel Hospital A-FIB Database (MIT-BIH). Twenty-three subjects were analyzed using subsections, and 45% of the RR-Intervals among the subjects were classified as A-FIB. ECG recordings for each subject were separated into subsections, collections of 25 RR-Intervals, and analyzed using Leave One Person Out Cross Validation (LOPO-CV). Volatility, an important predictor in LOPO-CV, was used to measure the dispersion of the length of the RR-intervals in seconds. Shannon, Approximate, and Sample Entropy were also used to classify A-FIB. These entropy features measured the irregularity of RR-Intervals, where decreased regularity suggested increased entropy values. Hyperparameter tuning on all tree-based methods and some non-tree-based methods was conducted to improve accuracy. The results of this study indicated that Sample Entropy was a strong predictor of A-FIB. Tree-based methods such as Random Forests and Cat Boosting had higher accuracies than non-tree-based models. If implemented in wearable heart rate monitoring devices like smartwatches, these results can aid in the preliminary diagnosis of A-FIB outside clinical settings.

UT2.2 *Harvesting-mediated emigration can affect community structure in a competitive system*

Savannah Humphries (Auburn University Montgomery (AUM))
shumphr6@aum.edu
Additional author(s):Jacob Garrett (AUM), Hamza Abusammour (AUM), Jerome Goddard II (AUM)

Trait-mediated behavioral responses (an indirect effect) to other species can affect population dynamics significantly. One example of such a response is modification of emigration probability, which has the potential to change species interactions and community structure. Habitat loss and fragmentation due to anthropogenic activities creates landscape-level spatial heterogeneity where remnant patches are often surrounded by a hostile matrix. Matrix composition or hostility is an important component of a landscape and can have profound effects on species movement and boundary behavior, persistence of a single species, and coexistence of interacting species. We model a system of two competitors dwelling in the same remnant patch surrounded by a hostile matrix. In this case, one competitor is being harvested (direct effect) and the other is not being harvested but is disrupted by the harvesting process, causing increased emigration (indirect effect). Thus, the second competitor exhibits a positive relationship between harvesting effort and emigration, i.e., harvesting-mediated emigration. In this talk, we will introduce our modeling framework and share some recent results demonstrating that community structure can be altered because of harvesting-mediated emigration and habitat fragmentation.

UT2.3 *Using Machine Learning to Predict the Number of Times Aid is Requested from a Local Nonprofit Agency*

Sydney Wilson (Converse University)
slwilson001@converse.edu

This project is focused on creating data-driven solutions for the Upstate Family Resource Center (UFRC), a nonprofit in Spartanburg County, SC. The UFRC serves as a central location for those in need to receive multiple types of aid with the goal of helping people regain their self-sufficiency. I will be discussing the results of using various machine learning algorithms to place clients into four unique groups based on projecting the amount of aid they will need to reach self-sufficiency. The accuracy of these models and future extensions will also be discussed.

UT2.4 *Exploratory Analysis of Playoff Overtime in the NFL*

Tyler Rielly (Coastal Carolina University)
tmrielly@coastal.edu
Additional author(s):Dr. Nicholas Pritchard, Coastal Carolina University

The National Football League (NFL) instituted a sudden death format to overtime in 1974. Since the first team to score wins, the coin toss winner most often chose to receive the ball. Thus, the winner of the coin toss gains an advantage in overtime. From 1994 - 2009, almost 60% of coin toss winners went on to win the overtime period. Due to the well documented advantage of winning the coin toss, the NFL fully implement a modified overtime system in 2012. However, due to the results of the playoff game between the Kansas City Chiefs and the Buffalo Bills in 2022, the NFL further refined the rules to the modified overtime system. The new rules took effect during the 2022-2023 season for playoff games only in an attempt to mitigate the impact of the coin toss when compared to previous overtime systems. Using the law of total probability and absorbing state Markov Chains, it is found that the most recent modification to the overtime system accomplishes this goal in theory in reducing the importance of winning the coin toss.

UT2.5 *Modeling Predator-Prey Relationships in the Presence of a Multi-Species Parasite*

Andalib Samandari (Georgia State University)
asamandari1@student.gsu.edu
Additional author(s):Daniel Dale, Undergraduate Student, Clemson University

Our ecosystem is held in a delicate balance by the relationships between different species. When human activity alters a habitat, the resulting changes in flora and fauna populations can have unexpected - and sometimes highly damaging - effects. We demonstrate this in the context of the predator-prey dynamic between the great blue heron and the Pacific tree frog, where environmental pollution from humans can increase levels of the multi-species parasite Ribeiroia ondatrae (or frog-mutating flatworm), which directly influences predation rates of frogs by birds. We use an original differential equations model, as well as an agent-based computer model, to examine the effects of varying parasite levels on bird and frog populations. Our differential equations model explores the effect of induced deformity in frogs on the parasite's ability to spread between hosts, and our agent-based model examines the parasite's effect on host population stability. Together, these models help us build a picture of parasitic disease and human influence on the ecosystem.

UT2.6 *Linear Models for Arrival Delay of Flights*

Michael Hendrix (Coastal Carolina Student)
mjhendr1@coastal.edu
Additional author(s):Dr. Lindsey Bell - Faculty Dr. Joseph Njuki - Faculty

Flight delays are something that nobody likes but everyone must deal with it. Whether your plane was late pulling or arriving, delays are inevitable. In this talk I will discuss how I used linear models to predict the arrival delay of a flight. Different types of linear models will be discussed, and later apply them in real-life data to propose the best model selected using different variable selection procedures

UT3.1 *Group Theory Structures in Bobbin Lace*

Riley Beam (Coastal Carolina University)
rebeam@coastal.edu
Additional author(s):Dr. Thomas Hoffman, Coastal Carolina University

Bobbin lace is a complex textile art that intricately weaves dozens of threads into a single piece. Upon examination, it becomes apparent why bobbin lace is able to be so elaborate while remaining structurally sound. This talk will discuss the group theory structures in bobbin lace by abstracting the textile to build a mathematical model as well as applications in pattern making through enumeration.

UT3.2 *REALIZABLE POSETS OF SOME MONOMIAL IDEALS*

Joseph Clark (Lander University)
joseph.clark4@lander.edu
Additional author(s):Trevor Leach, Lander University

A support poset is a useful algebraic tool for depolarizing monomial ideals to study the reliability of multistate coherent systems (Mohammadi et al., 2020). The support poset encodes the relationship between the variable of some polynomial ring and the minimal monomial generators of the ideal. It is known that a given poset may be the support poset of many square-free monomial ideals, while other posets are not realizable as support posets. However, a current classification does not exist for all posets. In this talk, we will discuss specific classes of posets that are realizable as support posets as well as classes of posets that are not realizable as support posets.

UT3.3 *Semigroups that are closed under differences*

Dmitry Demin (Georgia State University)
ddemin1@student.gsu.edu
Additional author(s):Dr. Florian Enescu, The Department of Mathematics and Statistics at Georgia State University

In commutative algebra, some classes of rings appear as subalgebras of polynomial rings generated over a field by monomials with support in an affine semigroup. Some special semigroups have properties that are reflected in intricate structures for the algebras. We will study three such classes of semigroups: semigroups that are closed under differences, closed under right differences, and, respectively, closed under left differences. We will present their properties, give examples, and study they structure for the case they are embedded in the nonnegative plane.

UT3.4 *Fibonacci Shapes and Patterns*

Angela Brobson (Converse University)
albrobson001@converse.edu

In this project, the Fibonacci sequence is graphed in the second and third dimensions in order to perform length, area, and coplanar calculations. These calculations result in patterns that can be seen throughout the sequence. It was discovered that the length between any two consecutive points represented by (f_n, f_{n+1}) and (f_{n+1}, f_{n+2}) is the square root of every other Fibonacci sequence value starting at f_1. Further, the ratio of two consecutive line segment lengths converges to ϕ. When graphing triangles with the sequence, it was discovered that the areas of the first ten triangles formed from the even numbers of the sequence equal 8, and the areas formed from the odd Fibonacci numbers seem to result in a new sequence entirely. Finally, it was discovered that ordered triples of the sequence are all coplanar. The recursive and closed form of the Fibonacci sequence are used.

UT3.5 *A Developmental Analysis of Problem-Solving Skills in an Introduction to Proofs Course*

Madelyn Parsons (Charleston Southern University)
mgparsons@csustudent.net
Additional author(s):Dr. Ryan Thomas, Charleston Southern University Dr. Emily Thomas, Charleston Southern University

The nature of learning mathematics changes dramatically in an introductory proofs course. This is the turning point in which students are encouraged to take their procedural knowledge and apply it in unfamiliar ways that enable them to use creativity to develop a broader scope of mathematical knowledge. We sought to analyze the development of problem-solving strategies throughout the course and pinpoint important factors for the success of the students. We analyze two types of learners in our study: "alien learners", who primarily focus on memorization and recitation, and "natural learners" who favor independent thought and conceptual understanding. Our sample group consisted of nine students enrolled in MATH 330: Discrete Mathematics. To assess student progress, we observed class daily, distributed an assessment at the beginning and end of the course to measure growth, and conducted interviews with the participants at various points in the course. Due to the limited sample, our primary focus is on the qualitative aspects of the data, in the development of four students who varied significantly. We will refer to them as Morgan, James, Emma, and Jack. Morgan struggled with anxiety and did not ask questions or engage in conversations when needed. She ended up failing the course and cited that she feels her inability to engage in conversation inhibited her learning. James struggled less with the material. However, James' natural ability meant that he was comfortable in his alien learning state and felt no need to adapt. He was even openly confrontational in discussions. Emma displayed potential for natural learning, but, when faced with challenges she leaned back into an alien learning state. Her reliance on procedure and reluctance to think abstractly may cause challenges for her in the future. Jack was a clear natural learner but was not particularly interested in the material or dedicated to the class. He grasped concepts quickly and adapted his own proof-writing style, whereas most students copy the style of the instructor. His understanding of the material was clear in the way that he spoke during discussions, and he contributed meaningfully to conversations. Overall, we found minimizing focus on procedure and an abundance of discussion to be key factors in student development. In this environment, students are forced to think about problems abstractly, which allows them to transition into the formal, conceptual world of mathematics more smoothly. In addition to the emphasis on abstract thought, this pedagogical approach allows for peer discussion and exposes the students to various ways of thinking about a problem. This not only strengthens their own grasp of the subject but allows them to fill in gaps in their conceptual understanding. Future studies will explore this approach to teaching an introduction to proofs course and analyze different tools to help students as they transition into the world of abstract thought.

7 Undergraduate Poster Session

UP.1 *On Topology and Projective Geometry - the Effect Curvature has on Accurate Rendering*

Lydia Mount (Gardner-Webb University)
lmount@gardner-webb.edu
Additional author(s):Dr. Robert Bass, Gardner-Webb University

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UP.2 *Classical Solutions of the Fornberg-Whitham Equation*

Georgia Burkhalter, Madison Waldrep (University of North Georgia)
gaburk1185@ung.edu
Additional author(s):Dr. Ryan Thompson, University of North Georgia

In this poster, well-posedness in classical solutions for the Fornberg-Whitham equation (FW) having quadratic nonlinearities is explored. We accomplish this objective by application of the fundamental theorem of ordinary equations to the nonlocal form of the FW equation in the Lagrangian framework.

UP.3 *Stability Analysis of Semi-Implicit Methods for Transport*

Kayli Hendricks (Augusta University)
khendricks@augusta.edu
Additional author(s):Anastasia Wilson (Augusta University)

This presentation will detail the stability analysis of semi-implicit methods (modified Backward Euler and Implicit Trapezoid) applied to the nonlinear transport equation utilized in the mathematical modeling of protein chromatography. Fully implicit methods are unconditionally stable, but given the semi-implicit nature of our equations, resulting from a lag of the adsorption coefficient, this stability expectation may change. Thus, this work seeks to determine whether these semi-implicit methods remain unconditionally stable.

UP.4 *How Confident Are You, Really? Simulated vs. Stated Confidence in One-Proportion Intervals*

Devyn Willey (Coastal Carolina University)
dwilley@coastal.edu
Additional author(s):Dr. Lindsey Bell, PhD- Coastal Carolina University

Confidence intervals estimate population parameters with a certain level of confidence. There are several options for constructing a confidence interval for the population proportion. With the help of statistical computing software, e.g., RStudio, one can simulate samples from a known population and compute different confidence intervals for a given problem simultaneously and significantly faster than computing them by hand. Using this computational power, we can compare the performance of the different intervals for the population proportion.

UP.5 *Modeling and Predicting Swimming Event Times for Elite Female Competitors*

Jovana Mitic (Converse University)
jmitic001@converse.edu
Additional author(s):Dr. Amanda Mangum, Mentor and Adviser

Elite swim programs want to help their swimmers peak at the correct time for competitions and perform at a top level for as long as possible. We collected data from elite female swimmers with long careers. Based on this data, we examined patterns in peak performance time and constructed a piecewise model to predict swim times based on age. Data from one event for elite swimmers, like Katie Ledecky, was then examined on an individual basis. The individual model was then compared to the general model built on the full dataset. Motivation, results, and the connection to training regimens will be discussed that could support coaches to maximize each individual swimmer's performance.

UP.6 *The Solvability of Magic Square Type Sliding Games*

Vigneswaran Madappan Chinnasami (University of South Carolina)
madappav@email.sc.edu
Additional author(s):Dr. Wei-Kai Lai, University of South Carolina Salkehatchie

A magic square is a square grid in which the numbers of each row, each column, and each of the two diagonals sum up to the same number. By subtracting 1 from each number and using the space of 0 as an empty block, we then create a magic square type sliding game. The goal is to arrange all the numbers, by moving one number at a time, to an increasing order form from left to right, and from top to bottom. Even though any reflection or rotation of a magic square is considered the same, each resulting sliding game after reflection or rotation is a new game. Since it is known that not all sliding games are solvable, in our project we analyze how the rotation and reflection affect the solvability of an existing magic square type sliding game.

UP.7 *An Agent-Based Modeling Simulation of DNA Self-Assembly and Design Strategies for Dipyramidal Graphs*

Ryleigh Henderson (Converse University)
rghenderson002@converse.edu
Additional author(s):Abbigale Outlaw - Converse University Dr. Jessica Sorrells - Converse University

DNA self-assembly is a type of molecular self-assembly. DNA self-assembly is a rapidly advancing field as DNA nanostructures created via self-assembly are useful in diagnosing and treating various illnesses, biomolecular computing, drug delivery, and biosensors. Due to the utility of DNA self-assembly, laboratories seek efficient methods to construct DNA complexes. In this case, efficiency translates into using the minimum number of different component molecules. To determine the minimum number, discrete graphs are used to model the DNA nanostructures and the self-assembly process. This work investigates the problem for the dipyramidal graph family within the framework of the flexible-tile self-assembly model. Dipyramidal graphs can be visualized as the skeletons of polyhedra which consist of two pyramids conjoined at the base. This work aimed to determine, under two different sets of constraints, the minimum number of tile and bond-edge types (i.e. component molecule types) necessary to construct dipyramidal graphs. This work also aimed to produce a program to model the self-assembly process in the agent-based modeling environment NetLogo. NetLogo, a programming language and integrated development environment (IDE), creates a visual output that is easier for diverse audiences to understand. Within the flexible-tile graph theoretical model we employ various mathematical techniques, such as valency sequences of graphs, linear algebra, and graph isomorphism. We determine that the minimum numbers of tiles and bond-edge types for dipyramidal graphs in the least restrictive scenario are two and one, respectively. Minimum numbers of tile and bond-edge types for odd sizes of dipyramidal graphs in the more restrictive scenario are also two and one, respectively. Using a graph theoretical model, minimum numbers of theoretical component molecule types were determined for dipyramidal DNA nanostructures. A program for a simple agent-based model of DNA self-assembly was programmed in NetLogo. This project is supported by a grant from the South Carolina INBRE funded by the National Institutes of Health National Institute of General Medical Sciences (P20GM103499).

UP.8 *Techniques for Solving Linear Programs including the Simplex Method and Extensions to Quadratically Constrained Quadratic Programs*

Savanah Crone (University of Tennessee at Martin)
savlcron@ut.utm.edu
Additional author(s):Larissa Renshaw (University of Tennessee at Martin)

The aim of our research is to solve linear programs using a new method derived from previous standard methods. Our investigation includes various cases based on the number of variables present in the system and the difficulty of the closed, convex feasible regions. We investigate common techniques including the geometric approach, the algebraic approach, and the simplex method. Along with these standard techniques, we explore a method that could be used beyond linear programs involving quadratically constrained quadratic programs.

UP.9 *Two Probabilistic Methods of Dealing With Missing Values in a Data Set*

Mark Britt (Francis Marion University)
msbritt444@gmail.com

In this research project, a model was constructed for a data set to describe and make predictions about the data. The model used was a Bayesian Network, which can be described as a number of variables linked together in a way that shows their relative independencies. Two methods of dealing with missing values in a data set were applied, and then compared. One standard method is to fill in all empty entries in the data set with random values. Our alternative method was to leave the entries with no values empty, and instead make a variable out of that missing information to use in the Bayesian Network model of the data. In particular, the number of missing values for each observation was recorded in a new column and used as one of the variables in the model. The models were then compared using a method called k-fold cross-validation.

UP.10 *Defining trajectories from recursive pairing between zeros and critical points of derivatives of random polynomials with independent roots*

Nickolos Monk (Appalachian State University)
monknm@appstate.edu
Additional author(s):Faculty Mentor, Noah Williams, Assistant Professor at Appalachian State University

We study the geometric properties of the zero sets of random degree-n polynomials (with roots chosen independently at random from the complex plane) and their non-trivial derivatives. Simulations readily reveal root and critical point "pairing" that persists under repeated differentiation, giving the appearance of distinguishable "trajectories." We propose an algorithm motivated by a classical proof of the Gauss–Lucas theorem to precisely define a set of "trajectories," and we present a preliminary investigation into their behavior. Our inquiry catches wind from recent results due to Galligo, Steinerberger, O'Rourke, Hoskins, Hägg, and others.

UP.11 *PIC Math Results*

Jordan Schmucker (Mars Hill University)
jordan_schmucker@mhu.edu
Additional author(s):John Moss and Claudia Chandler

This poster will summarize the results of the PIC Math (Preparation for Industrial Careers in Mathematical Sciences) course at Mars Hill University in Spring 2022. The industrial partner of the project was the Madison County (North Carolina) Transportation Authority, or MCTA. They are an on demand, non-profit organization that provides their services to county residence unable to get around by any other means. The project goal was to find the total cost per ride to the MCTA. As a team we worked with data from the MCTA and sorted through an issue of "empty miles"-sometimes seeing negative recorded values. Other factors we used to find the total cost, besides finding total miles, were maintenance costs of the vehicles, and the employee wages. Following the conclusion of finding the total cost per ride, we examine whether converting some of the vehicles to run on propane is cost effective or not.

UP.12 *Ramsey Theory on The Integers: The "L" Problem*

William Smith (Davidson College)
wismith1@davidson.edu

This paper aims to tackle the L number theorem, a problem in the field of Ramsey Theory. Formulated by Tibor Gallai and Ernst Witt and expanded upon by Ramsey Theorists such as Ron Graham and William Gasarch, the L number theorem states that for any integer k, there exists some n such that a k-colored n x n grid must contain a monochromatic L (a series of points (i, j), (i, j+t), (i+t, j+t) for some positive integer t). In this paper, we take a look at the upper and lower bounds for the smallest integer n such that a 3-colored n x n grid is guaranteed to contain a monochromatic L. We use various methods, such as counting intervals on the main diagonal, to lower the upper bound from 2593 to 1573. For the lower bound, we match the lower bound of 21 generated by Gasarch (2015) and look at ways to possibly improve it. Methods include finding underlying structures and distributions of numbers in the lower diagonal and cutting computation time in SAT solvers by fixing certain values.

UP.13 *An Exploration into the Efficacy of Virtual Homework in an Introduction to Proofs Course*

Madelyn Parsons (Charleston Southern University)
`mgparsons@csustudent.net`
Additional author(s):Dr. Emily Thomas, Charleston Southern University Dr. Ryan Thomas, Charleston Southern University

Recent research has explored the idea of whether the content in introduction to proofs courses matter, and it has ultimately found that the content is not as valuable as the development of problem-solving techniques and expansion of mathematical thinking. The transition from procedure to proof is unfamiliar in that it is less structured and allows more room for creativity. It has been concluded that students recognize "that their most influential instructors tended to give them the opportunity to discover and take ownership of the mathematics content" (Kercher, Álvarez). This is clearly a definitive moment in the education of mathematics students, yet the challenge lies in the actual development of a new method of mathematical thinking. Even the strongest procedural student often struggles with developing this new system of problem-solving, as this type of mathematics exchanges procedure for logic and flexibility. Procedural knowledge becomes secondary as students are "less compelled to remember and apply a 'correct' method after experiencing the flexibility of mathematics first-hand in an influential classroom environment" (Kercher, Álvarez). In classes such as college algebra or calculus, there has been a recent trend away from traditional handwritten homework to fully virtual homework. Drs. Burch and Kuo say that virtual homework, especially those provided to accompany textbooks, "offer the student instant feedback and online help while freeing the professors from the often cumbersome task of grading traditional paper-and-pencil homework." There has been much research conducted on the efficacy of such homework, but there has not been much emphasis on the impact virtual homework has on proof-level courses. Our project seeks to examine the impact of virtual homework in an introduction to proofs class. Our study consisted of eleven students enrolled in MATH 330: Discrete Mathematics. In our research, we have discovered that at least a mix between online and written homework aids students in their understanding of the material. This combination allows for a more stuctured side to homework—which reemphasizes concepts discussed in class, but also allows for creativity and freedom in proof writing through written homework. Continued research will discover the impact of this virtual homework in the overall success of the course and the influence this has for students as they advance to upper level courses.

UP.14 *Coordinates and Geometry in the Card Game EvenQuads*

Demmi Ramos (Lenoir-Rhyne University)
demmi.ramos@my.lr.edu
Additional author(s):MacKenzie Hulsey, Lenoir-Rhyne University

EvenQuads is a card game published by the Association for Women in Mathematics. It is similar to the game SET, but has cards whose symbol attributes each have four possible values instead of three. The goal is for players to look for special collections of four cards, called Quads. Based on the attributes and their values, one can assign the cards coordinates using the finite field $\mathbb{Z}_2$, making the entire EvenQuads deck equivalent to the vector space $(\mathbb{Z}_2)^6$, which is a model for the finite affine geometry $AG(6, 2)$. One can use these coordinates to study the algebraic and geometric properties inherent to the EvenQuads game.

In this poster, we will describe the rules of the game, show how to assign coordinates based on the attributes and their values, and explain some of the algebraic and geometric aspects of the game when viewed this way. We will then demonstrate how one can study these mathematical properties directly from the game without relying on preliminary coordinates, and how this gives an alternative and more general method for assigning coordinates to cards if desired.

This is based on joint work with MacKenzie Hulsey, supported by the Donald and Helen Schort Fund at Lenoir-Rhyne University.

UP.15 *A Numerical Study of expanded SEIR model for West Nile Virus Dynamics*

Jiwon Choi (Savannah State University)
jchoi1@student.savannahstate.edu
Additional author(s):Vinodh Chellamuthu, Ph.D., Utah Tech University, Abhinandan Chowdhury, Ph.D., Savannah State University

The rapid spread of West Nile Virus (WNV) has resulted in numerous infections and deaths among humans and horses and added significant economic burden to healthcare systems. The WNV propagates in nature through a cycle of transmission between birds and mosquitoes. A mosquito can become infected by feeding on an infected bird and spread the virus to people and horses through biting. The virus can also transmit from the female mosquito's eggs to her offspring - a phenomenon known as vertical transmission. Certain bird species tend to be more sensitive to WNV infection and frequently develop clinical signs that lead to fatal outcomes. On the other hand, once a bird recovers from WNV, it is immune to the disease. The bird has a high chance of passing its immunity down to its young, a process known as passive immunity. In this project, we have studied a Susceptible-Exposed-Infected-Recovered (SEIR) model to better understand the transmission dynamics of WNV in a mosquito-bird-human-horse community. The model depicts a system of first order nonlinear ordinary differential equations with 24 unknowns. It also allows one to incorporate vertical transmission and passive immunity - two hitherto overlooked phenomena. The purpose of this study is to determine how controlling the larvae mosquito population can affect the containment of WNV transmission in an entwined and expanded system of transmission. The numerical simulation is carried out by utilizing advanced solver in MATLAB to measure the dependency of effective treatments on various parameters over a long period of time. The results are further demonstrated by using graphical presentations.

UP.16 *The effect of the financial crisis of 2007 and 2008 on Germany's economy*
Muriel Schindler (Converse University)
mschindler001@converse.edu
Additional author(s):supervising professor: Dr. Joeseph Barrera
Leontief input-output tables provide a clear picture of a country's economy as it shows all the different sectors and what each sector provides to other sectors, as well as taxes, demand, and output. The "output" row that the OECD tables provide is equivalent to the production vector that you would get by solving the equation X = (I-A)$\hat{(}$-1) Y, where X is the production vector, (I-A)$\hat{(}$-1) is the Leontief inverse matrix, and Y is the demand vector.
Since the OECD website also provides the Leontief inverse matrix, I have decided to build the demand vector by adding the columns that include demand activities of the original input-output table to solve the above equation. In order to show the effects of the crisis, I will apply this method to five different years with a two-year gap to be able to show a trend in the production. To simplify the results I will combine several sectors of the production vector of the original OECD input-output table, so that my final production vector only includes 16 sectors while maintaining all the information it had before. That will allow me to identify an overall growth or recession as well as which sectors were hit the hardest or recovered the quickest after the crisis over the course of 10 years.

UP.17 *The Morphology of Z[Sqrt[n]].*
Hope Lissitz (King University)
hopelissitz@yahoo.com
What are the units, irreducibles and primes of the rings, Z[Sqrt[n], where n is a square-free integer? We answer the question for n equals 2 to 97. For example, (8+3Sqrt[7]) is a unit of Z[Sqrt[7]].

UP.18 *The Mathematics that Built the Great Pyramid*
Emily Thigpen (Francis Marion University)
emily.thigpen@g.fmarion.edu
The Great Pyramid in Egypt is a wondrous work of art, but also a beautiful mathematical construction. Sand, earth, and brick were not the only materials used in the constructions of the Great Pyramid. Likewise, sledges, rollers, and levers were not the only tools used either. When the Great Pyramid was built, the Ancient Egyptians used math as the most useful tool. Trigonometry, algebra, geometry, and the Pythagorean Theorem are math concepts that were used to shape the Great Pyramid. Mathematics meant a lot to the Ancient Egyptians, as it was the answer to many of the problems and questions they encountered in Ancient Egypt. The remains of the Ancient Egyptian pyramids, such as the Great Pyramid, are proof that mathematics was in its developmental stages during the ancient civilization time period. The Ancient Egyptians were the talented mathematicians who put the tool of mathematics to good use to build what are now recognized as mathematical masterpieces. Regardless of whether trigonometry, algebra, geometry, the Pythagorean Theorem, or any other mathematical idea was the key to the Great Pyramid's construction, the Ancient Egyptians unlocked it.

UP.19 *The Guide to Success in STEM*

Brianna Bradley (Francis Marion University)
`Brianna.bradley@g.fmarion.edu`
Additional author(s):Minerva R. Brauss, Research Advisor

Bandura describes self-efficacy as how well a person can execute a specific task to deal with various situations (Matsumori, Kazuki & Koike, Yasuharu & Matsumoto, Kenji). Self-Efficacy can also be described as the intrinsic motivation a person possesses to succeed and can be related to academic achievement such as retention and graduation rates at the post secondary level. Factors such as gender, race, finances, first generation status contribute to enrollment and retention of college students in STEM fields. Majoring in science, technology, engineering, and mathematical (STEM) fields promotes careers that have shown to be beneficial for people by creating job opportunities. It is important to provide students with the appropriate resources and opportunities to pursue and achieve their goals in STEM fields. Thus, this study will use national data statistics and surveys to determine if a strong sense of self-efficacy contributes to the retention of students in STEM by examining factors such as ethnicity/race, first-generation status, gender, and socioeconomic status.