# 103rd Meeting of the Southeastern Section of the Mathematical Association of America Abstracts of Papers and Posters 

March 14-16, 2024

Note: Talks are sorted first by session name and then listed in the order in which they appear in the program.

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## 1 Invited Speakers

Torsion Points of Elliptic Curves<br>Abbey Bourdon (Wake Forest University)<br>bourdoam@wfu.edu

An elliptic curve is a geometric object with an especially rich arithmetic structure. These curves are involved in applications that vary from Wiles' proof of Fermat's Last Theorem to secure web browsing, and the challenge of working with them has captured the interest of mathematicians from Weierstrass to Serre. Despite their complexity, elliptic curves and their basic properties may be defined using only highschool algebra, and many important questions can be stated with little additional background. For this reason, they are in a superb position to provide an accessible glimpse of modern mathematics.
This talk will introduce elliptic curves and then focus on questions related to a class of points known as torsion points. No special mathematical background will be required.

Beyond 1, 2, 3, ...: Counting the Familiar, the Less Familiar, the Strange, and the Impossible
Blayne Carroll (Lee University)
bcarroll@leeuniversity.edu
Enumerative combinatorics is a fascinating journey into the realm of counting. In this talk we will review and resolve some common counting problems and visit a few counting problems that are less familiar.
We will begin by revisiting the basics of counting, exploring permutations and combinations in familiar contexts. From there, we will venture into less familiar territory, tackling combinations with repetitions and additional restrictions. We will uncover the beauty and complexity of these counting problems, discussing binomial coefficients, extended binomial coefficients, and their applications.
As we continue to push our understanding of counting techniques, we will encounter enumeration problems that on their surface seem impossible to resolve efficiently. We will explore permutations and combinations with repetitions and restrictions and use the powerful technique of generating functions to unlock their secrets. Through this exploration, we will not only expand our understanding of enumerative combinatorics but also discover its profound implications in various fields. Join us on this journey as we move beyond $1,2,3 \ldots$ delve into the extraordinary world of enumerative combinatorics.

Connecting Traditional Math to a Modern Application: Discrete Probability and Fraud Detection
Richard Cleary (Babson College)
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The MAA's new journal Scatterplot (https://www.tandfonline.com/journals/usca20) was created, among other reasons, to encourage mathematicians to explore ways to connect traditional mathematics topics to important modern applications. These connections can help mathematics majors be prepared to enter careers in many fields, particularly data science. In this talk we present an example of this type of connection by discussing applications of Benford's Law. This is a surprisingly simple discrete probability distribution that has proven to be useful in various fields, particularly accounting. We will get some 'live data' on testing a hypothesis with Benford's Law and discuss how including applications in many of our courses can work to help students with both career readiness and mathematical understanding.

Fostering a pedagogy of belonging to promote learning in diverse student populations
Duha Hamed (Winthrop University)
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Nurturing a sense of belonging in the classroom lays a solid groundwork for effective pedagogy. Building connections between teacher and students, as well as among students themselves, is pivotal in fostering this sense of belonging. The term 'belonging' embodies my distinctive teaching style, shaped by a blend of personal and professional experiences. Specifically, my participatory learning-based approach cultivates a feeling of belonging among diverse student groups, such as those at Winthrop University, instilling in them the confidence that they genuinely belong. This not only enhances their self-assurance but also empowers them to actively engage with and comprehend mathematics. Additionally, if time permits, I'll talk about my rewarding experience mentoring students in summer research about generalizing distributions.

Share a Seat. Share a Cup.
Jeneva Clark (University of Tennessee, Knoxville)
dr.jenevaclark@utk.edu
The shortest distance between two people is a story. This talk tells stories. They need to be told. Our section was connected to the 1960 arrest of Dr. Martin Luther King, Jr., which we now know to have been a cusp in civil rights history. MAA member Bill Brodie, who was then a math graduate student at Atlanta University, led the student march and the integration of the lunch counters in Rich's Atlanta Department Store, with Dr. King by his side. The well-dressed and peaceful student activists chanted "Jail! No bail!" as they embraced imprisonment. Our own Bill Brodie traded his freedom for a jail cell bunk he shared with Dr. King for 7 days. Just months before that famous arrest, Bill Brodie, along with a classmate and 2 faculty from Atlanta University, travelled to the annual MAA-SE meeting, only to be turned away from the conference hotel. In that 1960 moment, our conference organizers had no way to overturn the hotel's segregation. They also had no idea that they were apologizing to a student who would soon change US history. It's not too late for us, to learn and to love. Bill Brodie, 89, will join us for an interactive audience Q\&A session following this talk.

## A Glimpse at the Horizon

Deanna Haunsperger (Carleton College)
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What do a square-wheeled bicycle, a 17th-century French painting, and the Indiana legislature all have in common? They appear among the many bright stars on the mathematical horizon, or, um, in Math Horizons. Math Horizons, the undergraduate magazine started by the MAA in 1994 publishes articles to introduce students to the world of mathematics outside the classroom. Some of mathematics' best expositors have written for $M H$ over the years; here is an idiosyncratic tour of the first ten years of Horizons.

## 2 Exhibitor Presentations

Introducing the Future of Homework: Unveiling WebAssign's Enhanced Platform for Students and Instructors
Gary Whalen (Cengage)
Gary. Whalen@cengage.com
Join us at this session to unlock the full potential of the WebAssign platform and discover how enhancements and features can elevate your teaching and empower your students to excel. Don't miss out on this opportunity to stay ahead in the world of online homework platforms! We look forward to seeing you there!

Engagement 8 Interactivity in the Classroom
Apryl Sweat, Cory Shafer (Pearson)
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Our presentation will explore innovative strategies to enhance engagement and interactivity in the classroom. From interactive platforms to gamified learning experiences, we'll delve into cutting-edge solutions that captivate students' attention and foster a dynamic learning environment. Join us to discover the latest trends and technologies shaping the future of education. Let's revolutionize the way we engage learners!

Bridging the Knowledge Gap and Encouraging Equity with Alta's Dynamic Remediation
Whitney Porter (Wiley)
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The mission of Wiley's affordable, accessible, and adaptive learning platform, Alta, is to bridge knowledge gaps and put achievement within reach for all students. In this presentation, we will explore how the power of adaptive learning can bridge prerequisite knowledge gaps and address state curriculum requirements without asking instructors to stretch themselves even further than they already do. Integral to this discussion will be an exploration of the qualitative and quantitative data about the success of Alta and adaptive learning as an equitable solution for students across all levels of math in higher education. We will give special attention to courses that are under review as a part of the Florida math redesign.

## 3 Special Session: Active Learning in Undergraduate Mathematics

ACT1.1 The Positive Impacts of Self-Assessed Homework Assignments in Calculus 1 Kristen Mazur (Elon University)<br>kmazur@elon.edu<br>Additional author(s): Carolyn Yarnall, California State University, Dominguez Hills

A key component of equitable grading is allowing students to reattempt problems without penalty. However, the process of providing feedback on assignments and regrading subsequent submissions is often prohibitively labor intensive. To balance fostering productive struggle with our own time constraints, we have created a Do and Review homework system in which students self-assess their assignments by identifying their mistakes and reflecting on what they learned from the mistakes. They earn full credit if their self-assessment is thoughtful and detailed. We conducted a study on the impacts of the Do and Review homework system on student learning in Calculus 1. Early results indicate that many students found that the Do and Review homework was beneficial to their learning and helped them understand the value of making mistakes. Moreover, grading these assignments is not time intensive. In this talk we provide further details on the Do and Review homework system and on the results of the study.

ACT1.2 Flipping the Script: A Flipped Model with LaTex Integration for ProofsIntensive Course
Catrina May (University of North Georgia)
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The ability to write solid proofs is an essential part of both communicating and understanding mathematics, and creating an environment where students are empowered to actively develop their proof-writing abilities is an important part of an effective mathematics curriculum. This talk will explore the implementation of a flipped classroom model with LaTex integration in two proofs-intensive undergraduate courses: Intro to Mathematical Proof and Abstract Algebra I. First, we will discuss the course setup, including examples of activities, assignments, and weekly schedules. Second, we will emphasize the importance of structured feedback within the model. Lastly, we will discuss several preliminary findings about the potential educational impacts of the model.
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ACT1.3 Activating Adjuncts' Active Teaching Opportunities
Grace McClurkin (Saginaw Valley State University)
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While we know that active learning is good for students, building an active classroom takes a lot of extra work! How do we create support and opportunities for adjuncts, who have less time than most, to build active classrooms? I will discuss how we are approaching this hurdle at SVSU, specifically for our adjuncts teaching our developmental math courses. Our approach can be split into two aspects: content and confidence. The first aspect involves building, curating, and distributing active-learning course materials, while the second involves discussion, exposure, and training about active learning.
This work has been tackled in conjunction with Dr. Joshua Mike and Christopher Servant of SVSU. It is supported by a Department of Education Title III grant and a Department of Education Rural Pathways for Student Success grant.

## ACT1.4 Modern Pen Pals in ODEs

Nicole Panza (Francis Marion University)
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Additional author(s): Amanda Mangum- Converse University
One concern we have as educators is whether our students can effectively communicate their mathematics. The Math Pals project addresses that through written, oral, and interpersonal communication. Our ODE classes at our respective universities created video lessons and supporting materials including homework and quizzes with solutions to send to the other university, "pen pal" style. This made our students the expert at their topic giving them an in-depth look at the material and encouraged them to think deeply about the details they needed to convey to an audience who has never seen this material before.

ACT1.5 Solving Geometry Problems using Calculus
Barrett Walls (Georgia State University) bwalls@gsu.edu
Additional author(s): Iason Rusodimos, Georgia State University
We take some challenging geometry problems from the MAA's Math Magazine and look at novel solutions using calculus instead of traditional geometric methods. We also discuss how to use these to help calculus students realize the versatility of calculus techniques.

ACT1.6 Forming and Maintaining a Vibrant Undergraduate Problem Solving Group Frank Patane (Samford University)
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This talk discusses the different types of undergraduate problem solving groups, benefits/drawbacks, likely issues one encounters, and what it takes to maintain such a group. This is done with a focus on the evolution of the Samford Problem Solving Group, and how you too can start (or improve!) such a group at your institution.

ACT2.1 Network Science in a First-Year Quantitative Reasoning Course Jenny Fuselier (High Point University)
jennyfuselier@gmail.com
We describe a quantitative reasoning course designed for Honors Program students at High Point University in North Carolina. The course is prerequisite-free and serves honors students from all majors across campus. The mathematical content of the course includes basic graph theory and some network science methods. Students also learn a new software package, practice writing to different audiences, and collaborate in teams on a multi-week project. Example student projects will be showcased, and both strengths and challenges of the course will be described.

## ACT2.2 Artificial Intelligence to Assist Problem Solving in Linear Algebra Jerzy Dydak (University of Tennessee) jdydak@utk.edu

I will present how I lead a discussion of a solution of a Linear Algebra problem in a classroom setting using AI bots. Most of them were tested in Spring 2023 and Fall 2023. Their details are described in my two papers (but reading of them is not necessary to follow my talk): 1. Artificial Intelligence to Assist Problem Solving in Linear Algebra DOI: 10.13140/RG.2.2.22087.27044 2. Artificial Intelligence and teaching of Linear Algebra DOI: 10.13140/RG.2.2.15727.20642 Time permitting, I will discuss the main ideas behind the two papers.

ACT2.3 Uhhhh... you know I'm a beauty and fragrance major, right?
Raju Bhusal (Savannah College of Art \& Design, Department of Liberal Arts)
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Additional author(s): Patrick Bahls, Savannah College of Art \& Design, Department of Liberal Arts
As members of a small math faculty at an institution focused solely on art and design, the task of making mathematics accessible, relevant, and maybe even fun to our students is a uniquely challenging one. This is especially true in our department's MATH 100 course, which many students use to fulfill the math and natural science requirement for majors ranging from painting and illustration to fashion marketing and jewelry design. Unsurprisingly, hands-on projects and student-centered activities encouraging inquiry-based exploration help students both to connect math with their respective creative disciplines and to overcome the math-related anxiety many of them bring to the classroom. We will both share projects we have used successfully in our MATH 100 classes, indicating how these projects can be adapted to suit the needs of students in general education math classes at a variety of institutions.

## ACT2.4 Active Learning in an Undergraduate Topology Course Shuler Hopkins (Sewanee: The University of the South) <br> sghopkin@sewanee.edu

When we think of introducing active learning techniques, we often envisage how these techniques will make our lower level courses more engaging to a broader audience. When we incorporate group work into a calculus course and hands-on projects into our introductory statistics class, we find that the class is more rewarding for everyone involved. In this talk, I will provide a proof (by example) that, if a teaching strategy is beneficial for our first year students, then it is beneficial for our senior math majors. I will talk about the active learning strategies that I introduced into my undergraduate topology course; what went well, what didn't go well, and what I would do differently next time.

## ACT3.1 Scaffolding for a Textbook-Based Inverted Classroom <br> Joshua Mike (Saginaw Valley State University ) <br> jlmike@svsu.edu

I will describe the design and implementation of an inverted discrete mathematics classroom built upon an OER text. Like most inverted classrooms, students prepare basic material before class and delve into higher-level thinking and problem solving during class. This course is the first upper-level math course for most of its students, who include mathematics, computer science, and computer engineering majors. Consequently, most of the students are still uncomfortable reading formal mathematics on their own. To help alleviate this strain, the course utilizes a series of "class prep summaries" to guide students in learning from a text. These summaries split content into easier "pre-class" concepts and harder "in-class" concepts to make expectations more transparent.
These classroom materials were developed with partial funding through a Title III Department of Education Grant.

ACT3.2 A small change that improved group learning in Calculus II

## Meredith Burr (Clemson University) burr3@clemson.edu

Following the advice of Building Thinking Classrooms in Mathematics, I have recently increased active learning in my Calculus II classroom through the use of "vertical non-permanent surfaces", i.e. whiteboards and blackboards. While I had incorporated group learning before, the first time I required groups to work out problems on whiteboards versus handouts, I was amazed by the significant increase in student engagement. These group activities can take more time and require modifications to my lessons plans, but the time is well spent. In this talk, I'll discuss my experiences implementing this strategy, my logistics and additional resources needed, student feedback, existing challenges, and future plans for scaling these methods to other Calculus II sections.

ACT3.3 Thinking Outside the Quiz: Exploring Alternative Assessments in LowerDivision Math Courses
Kelly Buch (Austin Peay State University )
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A quiz is a go-to assessment tool in many lower-division mathematics classes, but quizzes have their flaws. The usual routine of students scribbling down steps in order doesn't really get students to explain what's going on in their heads. We hope that acing a quiz means they've grasped the concepts, but that's not always the case. Indeed, a student could ace a quiz by memorizing what to write when without actually building any conceptual understanding which support the written steps. In this talk, I'll discuss two assessment methods I use in place of quizzes in my lower-division courses to address this issue. In these writing assignments and video solutions, students write out their steps as in a traditional quiz, but they also articulate their thought process and reasoning either in writing or out loud. I'll give examples of these assessments in lower-division courses, discuss strengths and weaknesses in implementation, and provide guidelines for when each assessment type might be superior to a quiz.

ACT3.4 Collaborative Creativity in Inquiry-Based Introduction-to-Proof
Amanda Lake Heath (Middle Tennessee State University)
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Creativity is recognized as an important 21st century competency and is integral to the work of mathematicians. The emergence of inquiry-oriented instruction at the undergraduate level has engaged students in more collaborative creative mathematical endeavors, yet research on mathematical creativity, and how to foster it, has been historically limited to the scope of independent work. To gain insight into student experiences of collaborative creativity in an inquiry-based course context, students responded to Retrospective Writing prompts throughout the course in which they described how they (and their group) functioned creatively during a proving exercise. These written narratives were qualitatively analyzed to produce themes describing student experiences. In this presentation, I will describe the major themes of moments within collaboration that made students feel creative and provide recommendations for how to foster creativity through collaboration in inquiry-based mathematics instruction.

ACT3.5 Enhancing Students' Active Learning Through Comprehensive Grading Rubrics
Yuerong Wu (Georgia State University) ywu31@gsu.edu
Grading rubrics are essential tools for guiding student learning and fostering improvement in educational settings. However, the lack of clear grading criteria often leaves students puzzled about their mistakes and areas for improvement. This presentation advocates for the necessity of providing comprehensive rubrics for each assignment, outlining grading criteria and deduction processes. I will present some real examples of how detailed rubrics stimulate students' active learning and enhance their understanding. Further, I will discuss the benefits of clear grading guidelines in empowering students and improving educational outcomes.

ACT3.6 Fostering Active Students' Participation in a Web-based Classroom Response System
Sutandra Sarkar (Georgia State University) ssarkar@gsu.edu
Deeper learning is facilitated when students have the opportunity for personal reflection about concepts through cumulative response systems. One effective strategy involves the implementation of web-based classroom response systems, which enable visualization and active participation. This approach fosters inclusivity within the learning environment and contributes to success in an introductory course through heightened engagement. In this presentation we will see together how anonymous student involvement helps all participants strive in a prerequisite course, e.g. Precalculus.

ACT4.1 Scaling Active Learning: Strategies for Success in Large Enrollment Courses Jack Ryan (University of Tennessee, Knoxville)
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Additional author(s): n/a
Managing active learning in large enrollment courses presents unique challenges for instructors. This conference talk will explore practical solutions to the logistical hurdles of implementing active learning in classrooms with over 100 students. By examining technological platforms and organizational methodologies, we'll uncover strategies that streamline the flipped classroom approach for maximum effectiveness. From leveraging online tools for collaboration to structuring group work efficiently, attendees will gain insights into creating a dynamic learning environment that engages every student. We'll discuss how instructors can adapt pedagogical techniques to suit the scale of their classes while maintaining student engagement and learning outcomes. Join us to discover how to navigate the complexities of large enrollment courses and unleash the full potential of active learning for your students.

ACT4.2 Visual Abstract Algebra: Puzzles and Pedagogy
Matthew Macauley (Clemson University)
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Group theory involves the study of symmetry, and its inherent beauty gives it the potential to be one of the most accessible and enjoyable areas of mathematics, for students and non-mathematicians alike. Unfortunately, many students never get a glimpse into the more alluring parts of this field because "traditional" algebra classes are often taught in a dry axiomatic fashion, devoid of visuals. In this talk, I will introduce some lesser-known groups via Cayley graphs, as well as some graphs that subtly do not define groups. Finally, I will showcase some "mystery graphs" that may or may not be groups, and leave the answers as fun puzzles. These can be explored and enjoyed by students and faculty alike, and make great exercises for an undergraduate algebra class.

## ACT4.3 Flipping Calculus at UNC Greensboro <br> Tom Lewis (UNC Greensboro) <br> tllewis3@uncg.edu

I will present on a new model used to teach calculus at UNC Greensboro that builds upon the learning materials developed during remote learning to flip the classroom. Prep problems are used to encourage students to engage with the material prior to attending class. Initial reflections are used for just-in-time feedback. Various sections of the coordinated course utilize the class time for learning activities, technology demonstrations, or study hall so that students can receive more help on homework. The flipped approach ensures we can keep a common schedule so that students in the calculus support course can receive targeted help with background content as it relates to the current calculus content. I will also talk about a new skills-based mastery approach that has ensured students engage with various campus support initiatives such as the Math Help Center. Qualitative results will be presented as well as some promising observations as we seek to meet students where they are and get them where they need to be without reducing the rigor of the course itself.

## ACT4.4 Graph Theory Games and Guided Self-Discovery Blake Dunshee (Belmont University) <br> blake.dunshee@belmont.edu

Few things can match the level of instant buy-in and classroom engagement that a stimulating challenge, puzzle, or game creates. This talk will focus on harnessing that immediate student engagement into mathematical self-discovery through graphtheoretical games. Through these exercises we can model the types of questions that mathematicians ask themselves while they approach problems. Then students can ultimately feel the satisfaction of solving a puzzle while understanding its connection to deep ideas in graph theory. These exercises have been used in many settings: camps for gifted high school students, quantitative reasoning students, and upperlevel undergraduate courses. We'll discuss how to engage all students in the discovery process.

ACT4.5 Empowering Active Engagement: Making Mathematics Meaningful Brian Pidgeon (Georgia State University) bpidgeon1@gsu.edu
What is your least favorite class? It may not be surprising to hear that many students initially respond that they dislike their mathematics and statistics course the most. As a result, making proper use of effective learning strategies has never been more important to help these students succeed. How can we accomplish this successfully? Active engagement that promotes a positive learning environment and promotes student input vastly changes the classroom atmosphere to encourage these students to become better learners. As learning styles are constantly evolving, this talk presents strategies and tips for making innovative changes in your mathematics classroom environment.

ACT4.6 Let's Play a Game! - How to use games to invite active learning Bill Shillito (Georgia State University)
wshillito1@gsu.edu
Who doesn't love a good game? Not only are they full of interesting mathematical properties, but they also provide an ideal environment for getting students actively involved in doing mathematics - sometimes without even realizing it. I will share some examples of how I have implemented mathematical games in my liberal arts mathematics class to create a student-centered environment and invite students to engage in group learning.

## 4 Special Session: Beyond Mathematics: Interdisciplinary Collaborations

## IDC1.1 Unraveling Plague Dynamics: Basic Modeling of Transmission Pathways in Black-Tailed Prairie Dog Populations <br> Vinny Jodoin (University of Tennessee) <br> vjodoin1@vols.utk.edu

Plague, a re-emerging disease according to the World Health Organization, poses threats not only as a natural outbreak but also as a potential bioterrorism weapon. Using the work from Webb et al. (2006). Classic flea-borne transmission does not drive plague epizootics in prairie dogs. Proceedings of the National Academy of Sciences of the United States of America. We investigate their model through an alteration to their model to mitigate the impact of plague outbreaks and apply a small change to understand how basic modeling practices are implemented using a story. This talk is ideal for individuals understanding basic principles of math modeling.

## IDC1.2 King Gilgamesh and the Big Breeches

Granite Clark (Tennessee Connections Academy)
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Additional author(s): Lara Qing-Bodiya with affiliations to Sequoyah Elementary School in 4th grade.
Hopefully, you're as excited for this year's MAA as we are (Lara Qing-Bodiya, 9, and Granite Clark, 14). In the Epic of Gilgamesh, one of the oldest works of literature in the world, the hero Gilgamesh is described via the size of his foot, leg and stride in ancient units of measurement. By first chasing the meaning of these units through limited scripture and then collecting data on modern day humans, we will be estimating the approximate height of Gilgamesh. By embarking on this project, we get practice in the collection and organization of data, as well as learning about curve fitting and basic regression analysis.

IDC1.3 Plato, Poincaré, \& Pythagoras: an initiation into esoteric mathematical astronomy
James Watkins (Appalachian State University)
watkinsjm1@appstate.edu
An investigation into the development of Western mathematics from Pythagorean ideals. The focus is on relating maths' position in the dichotomy of art/science through mathematical astronomy. We explore the philosophical trajectory of universal gravitation, aesthetics in maths, and the ubiquity of deduction. In particular, we want to answer if the Music of Heavens conjecture is enfact true.

IDC1.4 Mathematics in History: The Cases of Nicholas Saunderson and King George III
Matthew Cathey (Wofford College)
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This talk will introduce two interdisciplinary projects that the author currently has in process. The first is a collaboration with Dr. Chris Mounsey, a scholar of disability studies. Nicholas Saunderson was the fourth person to hold the Lucasian Chair at Cambridge. He developed a reputation for being the best teacher of what we now call calculus, at a time when it was only recently added to the curriculum. He was so widely loved by his students that a group of them pulled together some of his lecture notes to publish them after his death, for the benefit of his widow. Remarkably, Saunderson lost his eyes to smallpox as a toddler. Our book will tell his story: how did his blindness affect his life and career? And how did it influence how he understood and taught mathematics? The second project, to be undertaken with a student, involves the illness of King George III, who famously endured several breaks with sanity. We have daily records taken by his attending physicians; to my knowledge, our efforts will be the first attempt to use statistical methods to help shed light on his illness.

## IDC1.5 Tweaking Descartes on Light

Andrew Simoson (King University)
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To model light refraction, Rene Descartes imagined light was like a billiard ball rolling along a hardwood floor and then (non-normally) onto a rug. Such a model implies a single point of contact of the ball with its medium. But what happens, if we replace the ball with a rolling barbell, or better yet, a baton that skates along its two endpoints? If we further stipulate that should a baton endpoint ricochet upon a surface, then the baton rotates about its center until its other end also experiences a ricochet - we not only recover the laws of reflection and refraction, but also-and surprisingly so-a law of diffraction.

IDC1.6 Cryptologic collaborations with the arts and humanities
Christian Millichap (Furman University)
christian.millichap@furman.edu
Additional author(s): Sarah Archino, John Harris, Michael May, Grace Houser, Sophie Ngo, Sam Schaich, and Jordan White (all at Furman University).
Cryptology, the making and breaking of codes and ciphers, is a fascinating topic that offers ample opportunities to apply tools across the spectrum of undergraduate mathematics. Furthermore, cryptology has permeated popular culture in a variety of artistic forms: Sherlock Holmes stories, Dan Brown novels, the Kryptos sculpture, and many others. Thus, cryptology is a natural playground for collaborations with the arts and humanities. In this talk, the speaker will share an interdisciplinary creative project at the intersection of cryptology, puzzles, art, and storytelling, that is joint work with professors and students from mathematics, art, and art history. Questions and discussion post-talk will be most welcome!

IDC2.1 Partially Ordered Sets and Designing Classes in Object-Oriented Programming
Jeff Clark (Elon University) clarkj@elon.edu
In complex programs Object-Oriented Programming can be used to break down computations into classes: collections of attributes (the data) and methods that act on them (the code) which allow parts of the program to function independently. As features are added to them, individual classes can grow large and unwieldy. This talk will discuss how the theory of POSET's can be used to see when a class should be broken down into smaller classes to simplify the code.

## IDC2.2 The Mathematics of Redistricting: from Scholarship to Community Engagement to Undergraduate Research <br> Anne Catlla (Wofford College) <br> catllaaj@wofford.edu

Additional author(s): Alfie-Louse Brownless, Georgia Institute of Technology Following the 2020 census, communities across the United States underwent a redistricting process to redistribute voters across districts in a balanced way. This process can allow for the creation of districts that are designed to disenfranchise certain voters though gerrymandering. In this presentation, we will briefly describe some of the questions that are particularly relevant to mathematicians developing ways to detect gerrymandering and some of the questions that are particularly relevant to communities undergoing redistricting. We will then turn to the question of redistricting in South Carolina, which is particularly difficult to analyze due to the large number of uncontested races. We develop a district approximation method for replacing missing data from uncontested races and determine this approximation method's effectiveness.

IDC2.3 Stochastic Processes and Applications of Single Particle Tracking Keisha Cook (Clemson University)
keisha@clemson.edu
Biological systems are traditionally studied as isolated processes (e.g. regulatory pathways, motor protein dynamics, transport of organelles, etc.). Although more recent approaches have been developed to study whole cell dynamics, integrating knowledge across biological levels remains largely unexplored. In experimental processes, we assume that the state of the system is unknown until we sample it. Many scales are necessary to quantify the dynamics of different processes. These may include a magnitude of measurements, multiple detection intensities, or variation in the magnitude of observations. The interconnection between scales, where events happening at one scale are directly influencing events occurring at other scales, can be accomplished using mathematical tools for integration to connect and predict complex biological outcomes. In this work, we focus on building inference methods to study the complexity of the cytoskeleton from one scale to another. We rely on single particle tracking techniques based on stochastic models and explore long-term dynamics of the systems.
Additionally, stochastic processes can be used in many applications across the sciences. Each application requires a unique model, which is designed to exemplify the behavior of the system of interest. In this talk, I will explore some of these applications in the areas of intracellular transport in live cells (biology and material science), endosomal escape in cancer cells (bioengineering), and FRET models in protein dynamics (biophysics).

IDC2.4 Modeling the impact of temperature during nesting seasons on Loggerhead sea turtles
Suzanne Lenhart (U of Tennessee, Knoxville)
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Loggerhead sea turtles are a threatened species that nests on beaches along the northwestern Atlantic Ocean. Using data from Boca Raton nests, we investigated the relationship between air temperature and emergence success of hatchlings across multiple nesting seasons to better understand the potential impact of climate change on Loggerhead sea turtle populations. We predicted the effect of changing hatchling emergence success on the juvenile and adult populations. The results of a statistical model of emergence success feed into the dynamics during the nesting seasons of the eggs and hatchlings in a submodel on daily time scale. The submodel is connected to an age-structure model with two juvenile and one adult classes on a yearly time scale. We illustrate the effect of temperature changes across these life stages with this hybrid discrete model.

IDC2.5 Why Am I Learning This?
Lea Jenkins (Clemson University)
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Ten or fifteen years ago, how many of us would have been able to describe the workplaces of our graduates? Technological advances, along with the complexity of problems now facing society, require our graduates to have a solid understanding of the integration of mathematics with other disciplines. This requires us as faculty to continuously demonstrate the necessity of using mathematical tools and concepts in an holistic strategy to move solutions forward. In this talk, I will discuss several of the interdisciplinary problems I consider in my research, outlining the subproblems for various levels of students. I will also discuss the problems my modeling students have proposed for class projects, highlighting the desire of students to use mathematics to make a difference.

## IDC2.6 I Spy an Animal: Faculty-Student Collaborative Research in Ecology Rachel Grotheer (Wofford College) <br> grotheerre@wofford.edu <br> Additional author(s): Ben Sale (Wofford College), Eric McKenzie (Wofford College), Devin Ruppe (Wofford College)

In a college setting, especially at a liberal arts college, there are often untapped opportunities for the disciplines to work together. When those opportunities are explored, we can grow both as researchers and communicators as we learn to work with people in different disciplines. In this talk I will present two different research projects that I have worked with three different students on that were motivated directly by research done by a colleague of mine in Environmental Studies. I will discuss how the project ideas were developed through collaboration with my colleague, how the collaboration with the students started, and give an overview of each of the student's work. My goal is to demonstrate how our colleague's research in other fields can open up rich research opportunities and applications for both faculty and students in mathematics, and discuss how those collaborations could start.

IDC3.2 Making Meaning of Multidisciplinary Applications in Teaching Mathematics Sondra LoRe, PhD (STEM Program Evaluation, Assessment, \& Research (SPEAR))
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Abstract
This session engages the audience in research currently being conducted by four mathematics educators to discover the extent to which math faculty use multidisciplinary applications or real-world problems in their curriculum, the factors that are contributing to the current rate of usage, and what affordances and barriers they experience in these efforts.
Introduction
Student engagement in mathematics is linked to their perceptions of the relevance of the material (Williams, 2022). Interdisciplinary experiences enable the transference of skills and knowledge from one domain to another (e.g., quantitative skills to foundational biology courses), increasing the likelihood of success in those courses (Usher et al., 2010). Measuring the extent to which math faculty are incorporating applications into their curriculum and identifying the barriers to using such applications is a vital first step toward improving student engagement and success. To support interdisciplinarity and the transference of knowledge, the Quantitative Biology at Community Colleges ( $\mathrm{QB} @ \mathrm{CC}$ ) has spent the last five years forging collaborations between mathematics and biology instructors around the nation (Esquibel et al., 2023). The reasons biology faculty cite for minimizing the use of mathematics in their courses are well-researched. They include a need for pedagogical content knowledge and feelings of under-preparedness for effectively teaching interdisciplinary curricula (Corwin et al., 2019a; LoRe, 2019). This research extends what has been learned about biology faculty infusing quantitative skills into their curriculum to examine the inclusion of real-world applications from STEM disciplines into mathematics and statistics courses.
Research Questions
This additional research related to the QB@CC project aims to discover and evaluate the extent to which math faculty use applications or real-world problems from science in their curriculum, the factors that are contributing to the current rate of usage, and the barriers that prevent math faculty from incorporating more real-world applications into their teaching.
Research Methods We will employ an Exploratory Sequential Mixed Methods Research design (Creswell, 2015; Creswell, John, 2009), which includes a qualitative phase followed by the quantitative phase of research and culminating with an integration phase where the qualitative and quantitative phases are infused and reported. Phase 1: Exploratory Interviews
We began with interviews with mathematics faculty recruited by all four researchers at conferences and meetings over the last year. These included the QB@CC HHMI workshop, TMATYC, AMYTC, BioQUEST, HITEC, and NSF ATE conferences) and through snowball sampling (Coyne, 1997; Kemper et al., 2003) through connections to colleagues and social media recruitment. Interviews are being conducted via Zoom according to the IRB which includes a pre-survey. Following transcription of interviews, a collaborative, inductive and deductive approach to coding will be employed (Corwin et al., 2019; Merriam \& Tisdell, 2016).
Phase 2: inventory Survey
Following the analysis and reporting of the phase one interviews, the generated codes and themes will inform and contribute to the design of the phase two survey. We will employ a Delphi Method (Hartman \& Baldwin, 1995; Linstone \& Turoff, 2011) where QB@CC Steering Committee and Advisory Board members will be invited to provide feedback on the survey constructs, design, and dissemination (Robinson \& Leonard, 2019). Analysis of the survey with parametric and non-parametric measures (Slaney et al., 2008; Tabachnick \& Fidell, 2013) will infuse feedback and analysis from all

IDC3.3 Applied Math Powering Data Science Solutions to Problems in Industry, Society, and Academic Research
Rei Sanchez-Arias (Florida Polytechnic University)
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Applied mathematics provides strong underlying theories and methods that serve as the backbone for several data mining and machine learning algorithms. Coupled with significant advances in computing, its role has enabled the growth of interest and impact of data science solutions that enhance efficient knowledge extraction from structured and unstructured data in a variety of fields and applications. This talk discusses examples of the role of applied mathematics in proposed data science solutions to problems in industry, society, and academic research. Additionally, some examples for incorporating data mining and statistical learning concepts and methods in class projects will be presented.

IDC3.4 Groups, crystals, and symmetry
Matthew Macauley (Clemson University)
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Additional author(s): Maureen Julian (Virginia Tech, Materials Science and Engineering)
I teach abstract algebra in a unique "visual" fashion, and post all of my teaching materials, including hundreds of colorful pictures, on my website. Consequently, I get a number of emails about this from all over the world. One day, I got a message from the author of a popular crystallography textbook, which is also uniquely visual in nature. This turned into a fruitful collaboration about mathematical chemistry, and led to results that my collaborator described as being "shocking to have never been noticed in over 100 years of crystallography."

IDC3.5 Doorways to Calculus through Cryptocurrency
Andrew Penland (Eight Arms Nine Brains )
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Since 2018, many Decentralized Finance (DeFi) protocols have emerged, facilitating billions of US dollars' worth of transactions and the creation of over 40000 cryptocurrency tokens. Each DeFi protocol relies on algorithmic rules to calculate quantities like prices, fees, trade amounts. Many of these rules are understandable using basic ideas from Calculus. We will describe the connections between creating DeFi market rules and Calculus, including exercises and research questions that naturally arise.

IDC3.6 FEM in STEM: A Club to Connect Students from Different Disciplines Julie Barnes (Western Carolina University)
jbarnes@email.wcu.edu
Several years ago, some of our female students told me that they wanted to meet female students from other STEM fields, and that started the creation of our FEM in STEM club. The purpose of the club is to provide a supportive environment for women in science, technology, engineering, and mathematics. Anyone is welcome from a wide range of fields, including hard sciences, engineering, and computer science as well as health sciences, finance, and anyone who is simply interested in STEM fields. Sometimes men participate as well. In this talk we will address how we started the club, the kinds of activities we do, and how we have been able to fund events. An added bonus is that female faculty in these fields have had a chance to interact in ways we had not before.

IDC4.2 How an Industry Collapse Affects Community Mental Health: a Complex Contagion Model
Maggie Sullens (University of Tennessee)
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Additional author(s): David Finnoff (University of Wyoming), Katie Barkley (University of Wyoming), Nina H. Fefferman (University of Tennessee)
We adapt techniques from epidemiological models to examine the mental well-being of a community facing the collapse of the industry on which it is economically dependent. We consider the explicit case study of a fishing community facing the population collapse of its primary harvest species. By using an ordinary differential equation (ODE) susceptible, infected, recovered (SIR) framework with complex contagion processes, we track the movement of people through different mental health states as they shift their efforts to a more sustainable source of income by either changing harvest species or switching careers entirely. State changes occur due to either the direct observation of the declining harvest population or by interaction with other community members. We present three different possible scenarios for the decline of the harvest species, along with two effort functions to capture how fishermen may approach harvesting. We also present sensitivity of system behavior to different underlying mechanisms causing harvest species decline, potential conservation efforts, and the social connectivity of the community. We find that community connectivity has a meaningful impact on both the duration and transition dynamics, suggesting that interventions to support such communities may be most effective by focusing on socio-behavioral interventions.

IDC4.3 Modelling COVID-19 Dynamics Incorporating Vaccine Hesitancy Maruf Lawal (University of Tennessee, Knoxville)
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COVID-19 continues to pose a significant challenge to human lives globally with about 800 million confirmed cases and about 7 million deaths reported to the World Health Organization (WHO) globally as at today. Many efforts have been made to develop vaccines to control the spread of this virus and to this effect, about 14 billion doses of vaccines have been administered already worldwide. However, with the arrival of the COVID-19 vaccine, there is hesitancy and mixed reactions towards getting the vaccine. In this work, we propose and analyze a model of COVID-19 incorporating vaccine hesitancy using a system of ordinary differential equations. We will present an Ro analysis of the model and show the effects of key time-varying parameters using numerical simulations. We also compare our numerical results with data to better understand the factors contributing to the differences in hesitancy behaviors for selected countries of interest, including the United States, France, and South Africa.

IDC4.4 Predicting the Pandemic: Unmasking the Numbers Behind Social Distancing in South Africa's COVID-19 Response
Chidozie Williams Chukwu (Wake Forest University)
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Additional author(s): F Nyabadza, F Chirove, CW Chukwu, MV Visaya University of Johannesburg
The novel coronavirus (COVID-19) pandemic continues to be a global health problem whose impact has been significantly felt in South Africa. In this talk, we discuss a deterministic model to describe the impact of social distancing on the transmission dynamics of COVID-19 in South Africa. The model is fitted to data from March 5 to April 13, 2020, on the cumulative number of infected cases, and a scenario analysis on different levels of social distancing is presented. The model shows that with the levels of social distancing under the initial lockdown level between March 26 and April 13, 2020, there would be a projected continued rise in the number of infected cases. The model also looks at the impact of relaxing the social distancing measures after the initial announcement of the lockdown. It is shown that the relaxation of social distancing by $2 \%$ can result in a $23 \%$ rise in the number of cumulative cases, while an increase in the level of social distancing by $2 \%$ would reduce the number of cumulative cases by about $18 \%$. The model results accurately predicted the number of cases after the initial lockdown level was relaxed towards the end of April 2020. These results have implications on the management and policy direction in the early phase of the epidemic.

IDC4.5 Transmission of La Crosse Virus between Humans and Mosquitoes in Tennessee and North Carolina
Amber Young (University of Tennessee, Knoxville)
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La Crosse Virus (LACV) is spread by the bites of infected Aedes mosquitoes with most severe cases occurring in children under 16 years old. Children with LACV may also develop La Crosse Encephalitis. We use a system of ordinary differential equations with temperature and precipitation in some rates to model the transmission of LACV between humans and mosquitoes. We also use human case data from Tennessee and North Carolina.

## IDC4.6 Modeling the Spread of Crimean Congo Hemorrhagic Fever accounting for Tick Co-feeding <br> Jessica Kingsley (University of Tennessee- Knoxville) <br> jkingsl1@vols.utk.edu

Crimean Congo Hemorrhagic Fever (CCHF) is a tick-borne illness that affects animals and humans in countries across Africa and Asia. Ticks transmit this disease to susceptible animals by feeding on them and can acquire the disease by feeding on an infected host. Ticks feed on different hosts at different stages in their life cyclelarvae and nymphs feed on small mammals and birds, while adults feed on large mammals. Another aspect of the spread of CCHF is tick co-feeding. Co-feeding occurs when ticks feed on the same susceptible host within close proximity. If one tick is infected and the other is susceptible, the susceptible tick can become infected during co-feeding while the host remains susceptible. In this talk, I will present an ODE compartment model of the spread of CCHF that accounts for the life cycle of ticks and co-feeding. Preliminary equilibria and R0 results will be included as well. This work is a collaboration with MASAMU Advanced Studies Institute, an NSF funded research collaboration program with students and faculty in southern Africa.

## 5 Special Session: Mathematics and the Visual and Performing Arts

## MVP1.1 Surface-Tiling Curves

## Adam Rowe ()

email@adamrowe.com
This talk explains the process for constructing a space-filling curve, segments of which tile the faces of a regular polyhedron; describes its application to a polyhedron net; and illustrates some of its interesting properties with scupltural examples.

MVP1.2 Teaching Linear Functions with Linear Perspective
Katie Maxwell (Chester County High School)
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This paper explores the correlation between linear perspective and linear functions. The purpose of this paper is to determine if there is a relationship between math and art, and if that relationship can result in a deeper understanding for students. The paper presents a brief history of combining art and math, primarily during the Renaissance era; the benefits of combining math and art; a proposed lesson that covers both Algebra and visual arts' standards; and the benefits of the lesson from both a math and visual arts perspective. Examples from the lesson will also be presented.

MVP1.4 Graph Theory in Marching Band Rebecca Percy ()
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Marching band is an activity with which many today are familiar because of its popularity at half-time in American football games, but it also lends itself well to analysis using graph theory. Comprised of a group of musicians marching in formations on a field while playing music, a marching show bridges the musical arts, visual arts, and mathematics nicely. "Drill writing," the act of creating various formations and movements in a marching band show, resembles a complicated yet highly controlled set of car traffic patterns at an intersection, to which graph theory is often applied to find an optimal traffic light pattern.
This talk will: - Introduce marching drill charts and basic principles of drill writing - Examine a few marching formation transitions through the lens of graph theory at a traffic intersection - Use graph theory and integer programming to demonstrate how we might transition from one drill set to another via optimization - Discuss the implications of graph theory on drill writing software

## MVP1.5 Mathematic in Motion in Atlanta

Evans HARRELL (Georgia Institute of Technology)
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For several years mathematicians in Atlanta have worked with visual and performing artists to bring math to the public in novel and approachable ways. Shows and performances take place at the annual Atlanta Science Festival, on local stages, in schools, and in parks and neighborhood events. I'll describe some of these activities and lessons learned.

MVP2.1 Beauty in physics: math embodied through dance and movement Carrie Elliot, Annie Jennings () jenni142@gmail.com
Shapes and patterns are observed everywhere in nature, especially in mathematics. The beauty in prevalent factors like pi, the exponential and inverse logarithm, and the golden number have inspired curiosity in humanity. These patterns we witness everyday in life have major influences in the arts: a reminder of our evolutionary origins. Life unfolds in wondrous ways as art mimics life. The intersection of mathematics and performance is a compelling fusion of analytical rigor and creative expression. These interdisciplinary approaches have potential to generate novel insights and experiences.
Annie Jennings (DPT, MS in Physics) and Carrie Elliott (BS in Physics), will demonstrate mathematics-inspired aerial shapes and explain the physics behind them. We hope that you leave with a curiosity to find mathematics in art and expression.

MVP2.2 Lost in translation: exploring the connections music has to math
Lynn Sherman (University of Tennessee)
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In this talk we will explore the connections between mathematics and music theory. We often hear that there are lots of connections between mathematics and music. Even some of the concepts, we can feel it without being a musician. However, most of us does not know how to represent these concepts in formal way. During the talk we will focus on how music analysis techniques apply to an actual music piece. Then, we will translate music theoretical language into mathematics. This talk will cover variety of materials including sets, graphs, sequences, golden ratio, and more.

MVP2.3 How Numbers Weave: Exploring the Mathematics of Quilting Sydney Yeomans (University of Kentucky) seye224@uky.edu Additional author(s): Austin Fessler (University of Kentucky)
The University of Kentucky's Math Lab has made several mathematical quilts. Over that time, we've learned a lot about what math works best for this kind of visualization. We will talk about that, how we make our quilts, and the math behind a couple of our quilts.

## 6 Special Session: Recreational Mathematics

REC1.1 A Mixed Base Generalization of the 27 Card Trick Using Step Functions Ryan Loga (Southern Adventist University)
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The 27 card trick is a well known self-working card effect which involves sorting cards into piles so as to find a specified card at a predetermined location in a deck of 27 cards. Versions of this trick, including the even more well known 21 card trick, have been known for at last a couple centuries. Various explanations for the phenomenon have been offered. One of the more recent results involves using step functions to determine which set-ups yield an equivalent 21 card trick. In this talk we will discuss a generalization of the 27 card trick which uses any number of cards in a deck. This effect uses a non-unique mixed base representation to sort the cards. A proof will also be offered using the step function method. This is meant to be an accessible talk for mathematicians at all levels, regardless of their interests.

## REC1.2 Sprouts: A Game That Will Divide Friendships

Damian Belmontes ()
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Additional author(s): Demmi Ramos
Sprouts is a simple game that requires two players, a pencil, and paper. People of any age level can play this game without being aware of all the underlying mathematical patterns. Sprouts has been extensively studied and in our math seminar class, we explored this game by playing multiple matches, collecting data, making conjectures through observing patterns, and attempting to prove/disprove. This talk explores some of the mathematical variables involved such as the number of starting sprouts, dead sprouts, edges, moves, aspects of the graph, and more.

## REC1.3 Semiregular Tangles <br> Douglas A. Torrance (Piedmont University) <br> dtorrance@piedmont.edu

A Tangle is a smooth closed curve comprised of arcs of circles. The study of Tangles has been inspired by children's toys such as railroad tracks and the fidget toys of the same name. Most of the literature on Tangles has been devoted to square Tangles, where the arcs are all quarter-circles. However, any packing of the plane by congruent circles give rise to a family of Tangles. We introduce several of these families, each of which has been studied by a senior mathematics student at Piedmont University in recent years.

REC1.4 Quantum Connect 4
Shuler Hopkins (Sewanee: The University of the South)
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Connect 4 is a game in which two players alternate dropping tokens into a $6 \times 7$ grid; the first player to connect 4 in a row (vertically, horizontally, or diagonally) wins. In a perfectly played game of connect 4 , player 1 will always win; this result was first shown by Victor Allis in 1988. In this talk, we introduce a mathematical model for a quantum version of connect 4 which incorporates elements of superposition and entanglement. We then discuss the complexity of this version of the game and the existence of winning strategies under certain conditions.

REC1.5 A Study of the Tricolored Towers of Hanoi Nishant Chinnasami (University of South Carolina Salkehatchie)
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Towers of Hanoi is a popular game and has been studied by many people. Among all its variations, Chaugule studied the bicolor Towers of Hanoi problems in 2015. That inspired us to study the tricolor Towers of Hanoi. A Tricolored Towers of Hanoi game consists of a regular Towers of Hanoi 3-peg set, and three identical decks of regular Towers of Hanoi disks, but each deck in a different color. There are several ways to play with this new version, but all of them follow the same two rules of the regular version: (i) only one disk can be moved in each step, (ii) no bigger disk can be placed on a smaller disk. In this talk, we will introduce some ways to play this game, along with some patterns we observed.

REC1.6 Pi in the Sky - The Mathematics of Hot Air Balloons
Tabatha Rainwater ()
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Float away into a mathematical land of Hot Air Balloons. This talk will take place over two days. During the talk on day one, participants will learn the proper terminology and the typical process for flying a hot air balloon. We will explore the mathematics of ballooning through open-ended prompts with multi-media examples. Connections to Archimedes' and Bernoulli's Principles will be explored. Day two is highly dependent on favorable weather, in which we may have a live demonstration of inflating the "Pi in the Sky" hot air balloon at sunrise in the courtyard of Ayres Hall. Participants will be briefed on safe procedures, invited to participate in inflating Pi in the Sky , if weather permits, and encouraged to discuss the mathematics they experience in real time.

## REC2.1 The games and the mathematics of EvenQuads

Denise A. Rangel Tracy (Francis Marion University) rangel.tracy@fmarion.edu
EvenQuads is a new card game produced by the Association for Women in Mathematics (AWM) in celebration of their 50th anniversary. The double-sided card decks can be used to play a variety of mathematical games most of which are based upon a game originally created by Lauren Rose and Jeffrey Pereira. We will explore a variety of these games, such as Quad Collector and EvenBetter, as well as the mathematics underlying them. Additionally, we will highlight the ongoing research scholarship related to the mathematics of EvenQuads

REC2.2 3-D Carousels: How far do the horses run?
Jonathan Clark (Tennessee Wesleyan University)
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This presentation highlights a riddle posted by the Asheville Initiative for Math. The problem is generalized in several distinct ways, revealing unexpected connections to eigenvalue problems and star polytope classifications which naturally arise from these generalizations. Unexplored avenues of research into some of these generalizations are also discussed.

REC2.3 Hooked on Math with Topological Crochet
Emily McDonald (University of Tennessee, Knoxville )
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This talk will highlight the intersection of topology and crochet. This unique blend of mathematics and art not only demonstrates the accessibility and beauty of mathematical concepts but also encourages a hands-on approach to exploring abstract mathematical ideas through craft. Bring a hook, and we will begin to crochet a small Möbius Strip.

## 7 Special Session: Theory of Integer Sequences

INT1.1 How to walk to infinity on primes and square-free numbers
Joshua Siktar (University of Tennessee-Knoxville)
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Additional author(s): Steven J. Miller (Williams College), Tudor-Dimitrie Popescu (Brandeis University), Fei Peng (National University Singapore), Nawapan Wattanawanichkul (University of Illinois Urbana-Champaign)
In this talk we will explore the feasibility of constructing walks to infinity using only prime numbers, and using only square-free numbers. One of the main difficulties in constructing these walks is the computational complexity of factoring large numbers. To circumvent this difficulty, we produce and discuss stochastic models to approximate both types of walks, providing both theoretical remarks on the distributions of these models, and simulation results. In general, it is easier to walk to infinity on square-free numbers because, unlike the primes, these numbers have a positive density in the natural numbers.

## INT1.2 Chaos and Order in Ulam Sequences and Generalizations

Arseniy (Senia) Sheydvasser (Bates College)
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In 1964, Ulam defined a curious sequence that now carries his name. It starts with 1,2 , and then each subsequent term is the next smallest integer that can be written as the sum of two distinct prior terms in exactly one way. There are now countless generalizations of this basic idea in the literature, all of which seem to share the same curious property: the behavior of individual terms seems to be chaotic and unpredictable, but the aggregate behavior over many terms or many different sequences is very structured. Or, more accurately, it is conjectured to be structured based on extensive numerical evidence. In this talk, we'll give some results that chip away at some of these conjectures and let us say something definitive.

INT1.3 A Liberal Arts Project on Fibonacci-like Sequences
Ashley Johnson (University of North Alabama)
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One topic of MA 111 Math for the Arts at UNA is the Fibonacci sequence. We discuss interesting properties of the sequence, and then the students complete a project looking to see if these properties, or similar properties, hold for a sequence following the same recursive definition of Fibonacci with different starting values. In this talk we will introduce this project in detail.

INT1.4 Powerful and powered numbers
Tsz Ho Chan (Kennesaw State University)
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Powerful numbers are integers like 72 with exponents greater than 1 in their prime factorization. They behave sort of like perfect squares but with more irregularity. Powered numbers are smoothed versions of powerful numbers as introduced by Barry Mazur. In this talk, we will look at the distribution of these numbers over long intervals, short intervals, arithmetic progressions, as well as other aspects. There are still plenty of open questions and room for experimentation regarding these numbers.

INT1.5 Complementary Sequences, The Minimum Excluded Algorithm, Continued Fractions and Dynamical Systems

## Geremias Polanco (Smith College)

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The Minimum Excluded Algorithm is widely used, for instance, in combinatorial game theory and coloring problems in graph theory. It also has been used to generate complementary sequences, which are sets of sequences that partition the positive integers. In this talk we will show that applying the Minimum Excluded algorithm to generate complementary sequences is equivalent to prepending digits to the continued fractions expansion of irrational numbers. This gives rise to a dynamical system with an infinite family of fixed points that are quadratic irrationals.

INT1.6 The best two-term underapproximation by Egyptian fractions
Hung Viet Chu (Texas A\&M University)
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Recently, Nathanson studied underapproximation of a number $\theta \in(0,1]$ by Egyptian fractions or unit fractions, in other words. Specifically, the author considered the greedy algorithm, where at each step, the algorithm chooses the largest unit fraction strictly less than the remainder after previous steps. For example, to approximate 1 , we choose the first fraction to be $1 / 2$ because it is the largest unit fraction strictly less than 1 . As the remainder is $1-1 / 2=1 / 2$, the next unit fraction is $1 / 3$, then we have $1 / 7,1 / 43$, and so on. One main result by Nathanson is a method to determine, for a fixed $\theta \in(0,1]$, whether the greedy algorithm gives the (unique) best twoterm underapproximation. This method reduces the problem to checking a finite number of cases by computer, which he then demonstrated by considering $\theta$ with small $\lfloor 1 / \theta\rfloor \in\{1,2\}$; unfortunately, the complexity increases for bigger $\lfloor 1 / \theta\rfloor$.
We will see in this talk that for a special family of $\theta$, we can tell whether the greedy algorithm gives the unique best two-term underapproximation right away, and the family contains arbitrarily large $\lfloor 1 / \theta\rfloor$. In particular, the family must satisfy a certain threshold; if we go above the threshold, there are examples showing that the greedy algorithm does not produce the best two-term underapproximation. We will then discuss some open problems as well.

INT2.1 Benford's Law and Random Integer Decomposition with Congruence Stopping Condition
Steven J. Miller (Williams College)
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Additional author(s): Xinyu Fang, Maxwell Sun, and Amanda Verga
Benford's law is a statement about the frequencies that each digit arise as the leading digit of numbers in a dataset; base 10 we often do not see each digit equally likely, but instead see a digit of $d$ about $\log _{10}(1+1 / d)$ percent of the time. Many common integer sequences, such as the Fibonacci numbers (and more generally recursion relations) satisfy this bias. We prove that integer sequences resulting from a random integral decomposition process inspired from physical processes (which we model as discrete "stick breaking") subject to a certain congruence stopping condition approaches Benford distribution asymptotically. We also observe that the congruence stopping condition imposed is sharp; namely, once we let the stopping condition deviate above or below the given threshold, the resulting sequence fails to obey Benford's law.

INT2.2 On strongly pseudoperfect numbers and related inequalities
Joshua Zelinsky (Hopkins School)
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Additional author(s): Tim McCormack
We discuss strongly pseudoperfect numbers, numbers $n$ which are pseudoperfect and where they have pseudoperfect sum set $S$, such that $d \in$ if and only if $\frac{n}{d} \in$ $S$. We discuss how many properties of perfect numbers which do not apply to all pseudoperfect numbers do apply to strongly pseudoperfect numbers. A major role is played by the arithmetic-mean-geometric mean inequality.

## INT2.3 On 2-Near Perfect Numbers

Vedant Aryan (Hopkins School)
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"On 2 near perfect numbers," Abstract: A number $n$ is said to be $k$-near perfect if $n$ would be perfect as long as we ignore $k$ divisors. That is, there are divisors $d_{1}, \cdots d_{k}$ of $n$ such that $2 n=\sigma(n)-d_{1} \cdots d_{k}$. We discuss recent work on classifying all 2-near perfect numbers with two distinct prime factors.

INT2.4 On the unimodality of independence polynomials of highly non-regular trees Patrick Bahls (Savannah College of Art \& Design, Department of Liberal Arts)
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Additional author(s): Spencer Guess, Purdue University, Department of Mathematics (graduate student)
If $G=(V, E)$ is a graph, a set $U$ of vertices of $G$ is said to be independent if no pair of vertices from $U$ share an edge. The independence polynomial of $G$, denoted $I(G ; x)$, is the polynomial in which the coefficient $s_{k}$ on the term $x^{k}$ is the number of independent subsets of $V$ with cardinality $k$. It is a well-known conjecture of Erdos, et al. that the coefficients of the independence polynomial of a tree form a unimodal sequence, and unimodality has been proven for a number of families of trees, most of which have highly regular structure. In this presentation, we prove that if $G$ is a tree in which any two branch points are even distance from one another, then $G$ 's independence polynomial is unimodal, offering a dramatic generalization of earlier results.

INT2.5 Roots of polynomials with generalized Fibonacci coefficients
Ron Taylor (Berry College)
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Additional author(s): Jill Cochran, Berry College, Eric McDowell Berry College (retired)
In this paper we construct sequences of polynomials whose coefficients are generalized Fibonacci numbers. These generalized Fibonacci numbers arise from the two-term recurrence $s_{n}=a s_{n-1}+b s_{n-2}$ for arbitrary integers $a$ and $b$ with $s_{0}=0$ and $s_{1}=1$. These sequences have properties similar to the classical Fibonacci numbers including a relationship between the integers $a$ and $b$, and the limiting value of ratios of consecutive terms of the sequence, which we call $\varphi_{(a, b)}$. The sequences of polynomials arise from considering powers of the $\varphi_{(a, b)}$ and we show that each sequence of polynomials has a subsequence whose roots converge to $-\varphi_{(a, b)}$.

## INT2.6 Ucycles and Gucycles of Integer Compositions/Partitions

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Additional author(s): Anant Godbole, East Tennessee State University
A Universal Cycle, or Ucycle, is a cyclical representation of all the values of a combinatorial object; in the case that the object is all fixed-length words over a finite alphabet, these Universal Cycles are known as DeBruijn Cycles. For example, 11101000 is a DeBruijn Cycle of binary 3-letter words, and 00010111 is another. Chung, Diaconis, and Graham studied Universal Cycles for subsets, permutations and set partitions. Brockman, Kay, and Snively defined generalizations of Universal Cycles in which the objects were represented by graphs. These are called Graph Universal Cycles, or Gucycles; they proved the analog of DeBruijn Cycles for Gucycles. Cantwell, Geraci, Godbole, and Padilla studied Gucycles for the classes introduced by Chung, et al., and removed some restrictions on existence. In this talk, we will study Ucycles and Gucycles for a new class of objects, namely integer compositions and partitions, and prove results on their existence.

## 8 Contributed Paper Sessions

## CP1.1 Solvability by Irreducible Radicals over Finite Fields

## Matt Lunsford (Union University)

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A situation analogous to the classical casus irreducibilis exists in the context of irreducible cubic polynomials over finite fields if it is required that solvability by radicals means solvability by irreducible radicals. In this talk, we explore the situation for irreducible quartics and quintics over finite fields by use of the Capelli-Rédei Theorem.

CP1.2 Some nice quartics that aren't totally nice.
Nicholas Kirby (Austin Peay State University)
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Nice polynomials have only integer roots and integer critical numbers. Totally nice polynomials have all non-constant derivatives with only integer roots. It has been conjectured that there are no totally nice quartics with four distinct roots. A parameterization of a subset of nice, but not totally nice, quartics will be presented. We will consider the problem of showing that another such parameterization does not contain counterexample to the conjecture.

CP1.3 Predicting self-distributive algebraic structures through machine learning Dan Scofield (Francis Marion University)
daniel.scofield@fmarion.edu
Additional author(s): Sujoy Mukherjee (University of Denver)
Multiplication tables for algebraic structures, such as shelves and racks, are easily verified but expensive to enumerate. We show that a machine learning algorithm can accurately identify tables with the property of self-distributivity. In addition, given a partially filled table, a similar program can suggest completions which are valid examples of this algebraic structure.

## CP1.4 Characterizing Depolarization Posets that are Lattices <br> Ryan Gipson (Campbellsville University) <br> rgipson@campbellsville.edu <br> Additional author(s): Trevor Leach, Lander University

In 2020, Mohammade et al. introduced depolarization posets as an algebraic tool for depolarizing monomial ideals in $k\left[x_{1}, \ldots, x_{n}\right]$. While polarization of monomial ideals has been studied extensively in the literature, the authors discussed depolarization's utility in a variety of applications arising from its ability to reduce the number of variables of the ambient ring. In the 2020 paper, the authors stated that the depolarization poset was a meet-semilattice. A natural extension of this is to ask the question "for which ideals is the depolarization poset a lattice?" In this talk we will answer this question by providing a complete classification for which ideals produce a depolarization poset that is a lattice.

## CP1.5 Elasticity of Orders in Quadratic Rings of Integers <br> Jared Kettinger (Clemson University) <br> jkettin@clemson.edu

In this talk, I will discuss factorization in orders of quadratic rings of integers. Specifically, how a generalization of the Davenport constant for rings can be used to calculate the elasticity of orders of prime index in quadratic UFDs. I will also discuss how these results do and do not generalize in the Non-UFD case.

## CP1.6 Prime Labelings of Zero-Divisor Graphs

## Brad Fox (Austin Peay State University)

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Additional author(s): Christopher Mooney, University of Wisconsin-Stout The zero-divisor graph of a commutative ring $R$, denoted by $\Gamma(R)$, has the set of its non-zero, zero-divisors as the vertices with edges connecting any pairs that multiply to be zero. We investigate these graphs for particular classes of rings to determine the existence of a prime labeling, in which we label the vertices with distinct integers 1 to $|V(\Gamma(R))|$ so that any adjacent pairs have relatively prime labels. In this talk, we will develop prime labelings for zero-divisor graphs of some infinite families of rings, as well as show that there are infinitely many where a prime labeling does not exist.

CP2.1 Cesaro averaging and extension of functionals in $L^{\infty}(0, \infty)$
Pamela Delgado (Christian Brothers University)
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On the space of essentially bounded functions $L^{\infty}(0, \infty)$ we consider the Cesàro averaging operator $J f(x):=\frac{1}{x} \int_{0}^{x} f(t) d t$. We then extend the concept of integer iterates of Cesàro averaging $J^{n}$, to an operator of the form $J^{r} f(x)$, where $r$ is any positive real number and $f \in L^{\infty}(0, \infty)$. Our definition of fractional powers of Cesàro averaging is such that $\left(J^{r}\right)_{r>0}$ has the semigroup property.
Our work contains the following result: For any $f \in L^{\infty}(0, \infty), J^{r} f(x)$ has a limit at infinity for some $r>0$, if and only if $J^{s} f(x)$ has a limit at infinity for any $s>0$. In this case, the limit values are all the same. We present a strong quantitative version of the special case where $0<r \leq 1$ and $s=1+r$.

## CP2.3 A Study of the Regularized Gardner Equation

Pamela Guerrero (University of Tennessee at Martin, University of Memphis)
pguerrer@utm.edu
This study centers around the regularized Gardner equation. I show for initial data in the $L^{2}(\mathbb{R})$ based Sobolev space with $s>1 / 3$ the corresponding initial problem is well posed in the sense of Hadamard. That is, there exists a unique solution to the initial value problem that depends continuously on the initial data.

## CP2.4 A Mathematical Model for The Dynamics of Ovarian Cancer In Ghana Ebenezer Acquah (University of Tennessee, Knoxville) eacquah@vols.utk.edu

The most lethal cancer of the female reproductive system is ovarian cancer. HighGrade Serum Ovarian Cancer (HGSOC), which accounts for the majority of ovarian disease cancers, is the disease's leading cause of death. Ovarian cancers are increasingly being recognized as a family of nonuterine tubo-ovarian cancers rather than a single disease. This talk shall address the dynamics of ovarian cancer using a mathematical model. In this talk, we shall address the population of cancer in three stages: proliferating, quiescent, and dead cells.

## CP2.5 A Likelihood Approach to Filtering for Advection Diffusion Processes Johannes Krotz (UTK) <br> jkrotz@vols.utk.edu <br> Additional author(s): Juan M. Restrepo (ORNL)

A nonlinear differential equation is used to build the model, and the system is linearized using the Jacobian. The model's stability was ascertained using simulation study and mathematical analysis tools, such as identifying equilibrium points and stability analysis of the equilibrium points, proving the positivity of the solutions so that they are biologically meaningful, and so on. The Hartman-Grobman Theorem will help to achieve stability. In addition, we raised an unanswered clinical open question.

CP2.6 Numerical methods for solving Nanofluid flow problems
Ramjee Sharma (University of North Georgia, Oakwood, Georgia)
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Additional author(s): 1. Anil Devarapu, University of North Georgia, Oakwood, Georgia, Anilkumar.Devarapu@ung.edu ; 2. Dipendra Regmi, University of North Georgia, Oakwood, Georgia, Dipendra.Regmi@ung.edu
In this presentation, we will introduce a numerical method designed to address fluid flow problems, encompassing both the Blasius and Sakiadis problems. We begin by outlining our approach to solving these foundational problems. Subsequently, we extend our investigation to a more generalized framework, introducing the parameter $\lambda$. This parameter serves to delineate the boundaries of the Blasius and Sakiadis problem domains. Within the spectrum of $\lambda$ values $(0 \leq \lambda \leq 1)$, we explore scenarios involving the simultaneous motion of the flat plate and the free stream. Our aim is to provide a comprehensive understanding of the fluid dynamics under consideration. Finally, we expand upon these methodologies to tackle the complexities of nanoparticle-induced flow within the contexts of both Blasius and Sakiadis problems.

## CP3.1 Impact of Credit Risk in the Field of Lending Industry

Monsuru Olalekan Durojaiye (Austin Peay State University)
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Additional author(s): Ramanjit K. Sahi
Credit risk plays an important role in the lending industry. In my research, I am focusing on the insights from the borrowers' financial positions and loan performance using statistical data analysis. Initial findings have shown that there is a relationship between debt-to-income ratio (DTI) and loan defaults. Furthermore, risk-based pricing (RBP) would be estimated to mitigate the risk of default. All this will be of immense importance to loan officers, risk analysts, and other stakeholders involved in the lending process to make an informed decision.

## CP3.2 Interplay of Economic Forces and Geopolitical Factors in Oil Pricing Olayinka Ugwu (Austin Peay State University) <br> ougwu@my.apsu.edu <br> Additional author(s): Ramanjit K. Sahi

In a world where a political whisper in the Middle East can send gasoline prices spiraling, understanding the link between US crude oil and global events is vital. This research looks at the complex ballet of economics and international affairs in shaping the oil market and predictable rhythm. Specifically, we have applied regression analysis to study this interplay between economic forces and geopolitical factors in US Oil prices. Our analysis revealed a triumvirate of economic forces holding significant sway over oil prices. Booming U.S. crude production, a robust Real GDP, and a strong dollar all emerged as potent predictors of the West Texas Intermediate spot price. This reaffirms the long-held belief that oil prices are not solitary dancers but rather part of a complex economic ballet, where supply, activity, and currency all have their steps to perform.

## CP3.3 Cryptosporidiosis Forecasting in Tennessee Megan Oelgoetz (Austin Peay State University) moelgoetz@my.apsu.edu Additional author(s): Ramanjit Sahi

Extreme rainfall has an established positive correlation with climate change. This project will model gastrointestinal illness in Tennessee from past data taking into consideration rainfall data to evaluate the predictive power of increased rainfall and gastrointestinal illness incidence. Historical data of monthly cases of cryptosporidiosis, a gastrointestinal illness caused by contaminated water, will be evaluated with contemporary historical rainfall data from the years 2012 to 2021. The models used will be methods of time series analysis, specifically Seasonal Autoregressive Moving Average (SARIMA) models and SARIMA models with rainfall as a covariate (external regressor) to evaluate if this improves the model fit. Models will also be used to forecast future incidence of gastrointestinal illness. Findings are that including precipitation as a covariate in the best SARIMA model for cryptosporidiosis cases does not improve the model. Further testing of other possible environmental covariates is needed.

## CP3.4 Using Gaussian Elimination and the Null Space Method to Balance Chemical Equations

Zachery Keisler (Saluda High School)
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One of the fundamental problems in chemistry is balancing chemical equations. By examining this problem's structure more closely, we find that it shares many of the same properties as one of the fundamental problems in mathematics: solving systems of linear equations. With the problem of balancing chemical equations sharing many of the same properties as the problem of solving systems of linear equations, one can deduce that several techniques commonly used for solving linear systems can also be used in balancing chemical equations. One of these techniques that can be used for solving linear systems and balancing chemical equations is Gaussian elimination, which is found within the field of matrix theory. This method relies on turning the balancing chemical equations problem or the solving linear systems problem into a matrix. In this presentation, we will examine how Gaussian Elimination can be used to balance chemical equations while also exploring a similar method of balancing chemical equations found within the field of mathematical chemistry known as the Null Space method.

## CP3.5 A physics-inspired Bayesian method for learning molecular dynamics Daniel McBride (University of Tennessee, Knoxville) <br> dmcbrid1@vols.utk.edu

To quantify molecular dynamics in a principled way, we construct a statistical model and use a Bayesian learning method inspired by classical mechanics. We seek to sample from a high-dimensional non-standard target probability distribution known only up to a multiplicative constant. This target distribution encodes probabilities associated to the molecular dynamics. The method leverages gradient information of the target distribution to improve on standard Monte Carlo algorithms used in Bayesian inference. We describe the novel methodology for the estimation of molecular dynamics from experimental data.

## CP3.6 Statistically Shipwrecked: Scurvy makes a case for statistical methodology Troy Riggs (Union University) triggs@uu.edu

A presentation of the British navy's ongoing failure to recognize the effectiveness of citrus in sailors' diets as a means of curing and preventing scurvy, dramatically demonstrates the human cost in suffering and death when statistical principles are not implemented. For nearly two hundred years (c.1600-1800), medical officers and ship captains were unable to convert anecdotal accounts into sufficiently compelling evidence. Through this story, statistics students witness the need for controlled experiments, large samples, detailed records, and effective presentation of data in challenging established medical dogma and military health protocols.

## CP4.1 Increasing Student Success in Functions and Their Applications with the Development of a Corequisite Support Course <br> Elizabeth F. Lewis (Greensboro College) <br> elizabeth.lewis@greensboro.edu

I will discuss the design, implementation, and evaluation of the corequisite support course for Foundations of Functions and Their Applications at Greensboro College. This course was developed as a way to increase student success in our STEM gateway mathematics course as an alternative approach to the traditionally required remediation prerequisite intended for students with low mathematics placement. As an evolving pair of courses, I will introduce the initial design plan, lessons learned from successes and failures of that design, and impactful modifications that have been made to more completely meet student success goals.

## CP4.2 Teaching a Student-Friendly Liberal Arts Math Class Steve Nimmo (Piedmont University) <br> stevenimmo@piedmont.edu

Liberal Arts math classes are typically populated by students who do not like math, are not good at math, and are only in the class because it is the least offensive math course offered to meet the quantitative literacy general education requirement. In this talk, I will present several things that I do in my Liberal Arts math class that alleviates many of the concerns students have at the beginning of the semester and helps the students to complete the course successfully.

CP4.3 Hope, Fear, and Success: Reflective Writing Assignments in Lower-Level Math Courses
Jennifer Aust (The University of Tennessee Southern) jennkaust@gmail.com
The author will present a series of reflective writing assignments for lower-level mathematics courses focused on students' hopes and fears about their math classes. The initial assignment, given during the first week of classes, asks students to answer three questions about their hopes, fears, and strategies for success in the course; follow-up assignments ask them to revisit those themes later in the course. This series was implemented in two different courses in Fall 2023 and is being implemented in one course in Spring 2024. The presentation will highlight differences in the follow-up assignments, including changes made for Spring 2024 based on results from Fall 2023, and future plans for further improvements to the assignment series.

## CP4.4 The High School to College Transition: An Early Case Study of University High <br> Emily McDonald (University of Tennessee, Knoxville) <br> vng393@vols.utk.edu

This talk will focus on the scaffolding of mathematics education between high school and college through the University High school program, a partnership between Hamilton County Schools and the University of Tennessee at Chattanooga that began in Fall 2023. This presentation will explore the practical applications of this program by demonstrating its commitment to developing critical thinking and learning skills in mathematics among 11th-grade students transitioning to college mathematics in their 12th-grade year. The overall goal of the program is to be a college pipeline for high school students who might not have considered a four-year university education or who are seeking a more personalized educational experience. The presentation will also highlight the program's role in promoting educational equity and access without cost to students and families. This talk will focus on the program's strategies in the mathematics courses being offered in 11th grade to support students who are transitioning to a college mathematics course. This early case study will offer insight into practices for supporting students' mathematical journey from high school to college.

CP4.5 Constructing a corequisite for precalculus: the virtues of collaboration Jeffrey Lawson (Western Carolina University)
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Additional author(s): Daniel Best and Sibley Bryan, Western Carolina University
As part of our department's reform of mathematics service courses, we replaced a conventional college algebra course with a precalculus algebra course coupled with a corequisite. To create a corequisite for precalculus that is responsive and relevant yet still addresses the learning outcomes necessary for success in the path to calculus, we brought in many partners across campus, including some unlikely choices. In this presentation, we'll share our journey, as well as our outcomes, both expected and unexpected.

## CP4.6 Algebraic Structures on Graphs Joined by Edges

Daniel Pinzon (Georgia Gwinnett College)
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A graph is a finite number of points connected by edges. It can be used, for example, to model a computer network, business competitiveness, organic molecules, social media, etc. You can represent this graph as an adjacency matrix. We can create algebraic structures whose "multiplication" is the join of two graphs, in a one-to-one manner, by edges. This presentation will explore the effect that combining graphs has on the determinant. For example, what are the properties of a graph needed so that when joined to any graph, it does not change the determinant? We will explore and prove algebraic structures, properties and patterns using linear algebra, abstract algebra, and mathematics software.

## CP5.1 The Mathematics of Cycling

Thomas Schnibben (Francis Marion University)
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Many factors contribute to success in cycling: power output, aerodynamics of both the bike and the cyclist, weight of the bike and cyclist, ability to draft other cyclists, course, weather, and many others. In this talk we will investigate the methods of modeling these factors in an attempt to minimize time required to complete a given course. In particular we will focus on cyclists competing in an individual time trial, where drafting is not allowed. The relationship between power output, speed, and aerodynamics will be qualified in an attempt to determine which factors should be focused on to create the best possible outcome for a cyclist.

## CP5.2 Penalized Poisson Regression with Referee Index for a Professional Soccer Team

Mingwei Sun (Samford University)
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Additional author(s): Thomas Reinke, Baylor University
In this paper, a penalized Poisson regression model which can perform variable selection and regression coefficients estimation simultaneously was implemented to investigate which variables affect the performance of a professional soccer team. A new referee index was proposed in the model to demonstrate the influence of a referee on the team. The model was illustrated by an example of the Manchester City F.C. team in the English Premier League. It can select the essential variables that affect the number of goals in a game of the team. A testing result showed a superior performance of our new model compared with the regular regression models. The new model can also be extended to the analysis of the performance of a team in other fields of sports.

## CP5.3 The Structured Coordination Game with Neutral Options <br> John McAlister (University of Tennessee - Knoxville) <br> jmcalis6@vols.utk.edu <br> Additional author(s): Nina Fefferman (University of Tennessee - Knoxville)

 Coordination games have been of interest to game theorists, economists, and ecologists for many years. Approaches for understanding the coordination game with discrete structure have been limited in scope, often relying on symmetric reduction of the state space, or other constraints which limit the power of the model to give insight into desired applications. In this paper, we introduce a new way of thinking about equilibria of the structured coordination game with neutral strategies by means of graph partitioning. We begin with a few elementary game theoretical results and then catalogue all the Nash equilibria of the coordination game with neutral options for graphs with seven or fewer vertices. We use simulation to consider larger graphs to extend our observations and form the basis for proposing some conjectures about the general relationships among edge density, cluster number, and consensus stability.
## CP5.4 Quadratic Voting: Overview, Attacks, and Extensions <br> Andrew Penland (Eight Arms Nine Brains) <br> andrewpenland@gmail.com

In recent years, a number of new voting and funding systems have been proposed and put in use. One well-known example is Quadratic Voting, initially proposed by Weyl, Heitzig, and Buterin. The mechanism has been utilized in organizations ranging from The Democratic Party of Colorado to the cryptocurrency funding pool GitCoin. We discuss attacks, metrics, and investigations related to this and related mechanisms.

## CP5.5 p-Norm Approval Voting <br> Jessica Sorrells (Converse University) <br> jessica.sorrells@converse.edu

Additional author(s): Hari Nathan (University of Rochester), Michael Orrison (Harvey Mudd College), Katharine Shultis (Gonzaga University)
Approval voting is a well-studied voting procedure in which each voter chooses a subset of the candidates of which to "approve", and the candidate chosen most often is then declared the winner. Alternatively, approval voting can be described as each voter giving 1 or 0 points to each candidate, with winning candidates being those with the most points. Recently, there has been interest in satisfaction approval voting and quadratic voting, both of which temper the points a candidate receives from a voter with regard to how many other candidates were approved of by that voter. In fact, approval voting, satisfaction approval voting, and quadratic voting can all be viewed as each voter submitting a vector of 0's and 1's which is then normalized using different norms $(p=\infty, p=1$, and $p=2$, respectively). This suggests a family of voting methods generated by varying norm $(p)$ values. Here, we examine to what extent changing $p$ affects outcomes and satisfaction of some typical properties used to justify voting methods.

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CP5.6 Intravascular Stent Drug Release & Transport - Model, Analysis and Simula-
    tion
    Lucy Jiang (University of Tennesee-Knoxville)
    tjiang12@vols.utk.edu
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About 1 in every 20 adults aged 20 or older in the US are affected by coronary artery diseases. Intravascular stents, in particular drug-eluting stents, have become a popular alternative treatment to traditional open-heart surgeries, due to the minimal invasion and cost-effectiveness. Since the drug concentration in the arterial tissue needs to be kept within a therapeutic window, it is of great interest to study the drug release and absorption profile. Our work is based on a multi-physics coupled model. We prove that the PDE system is well-posed, and present the numerical simulations and analysis.

CP6.1 Varying Patterns: A Look at Generalized Delannoy Numbers Modulo Prime $p$
William Griffiths (Kennesaw State University)
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Additional author(s): Steven Edwards, Kennesaw State University
The Delannoy numbers form a doubly recursive integer sequence that have many uses and enumerations. In particular, they form very interesting patterns when viewed modulo odd prime p. By varying a parameter in the Delannoy number formulae, we introduce a family of doubly recursive integer sequences, the Generalized Delannoy numbers. We then examine this family modulo odd primes p and explore the resulting patterns.

## CP6.2 Counting Forests and Complete Graphs <br> J.C. Price (Georgia Gwinnett College) <br> jprice12@ggc.edu

In this talk, we would like to explore the number of spanning forests in a graph with a fixed set of vertices in each tree. In particular, we will introduce a formula for counting such forests and apply it to a complete graph. This will lead to an application of Cayley's formula for complete graphs, which will simplify this formula and lead to some further questions. (This work was done in collaboration with Daniel Pinzon and Daniel Pragel at GGC.)

## CP6.3 Counting Spanning Forests Using Kirchhoff's Theorem <br> Daniel Pragel (Georgia Gwinnett College) <br> dpragel@ggc.edu <br> Additional author(s): Daniel Pinzon and J.C. Price from Georgia Gwinnett College

A spanning forest of a graph $G$ is an acyclic spanning subgraph of $G$. When the spanning forest has a single connected component, it is referred to as spanning tree. A well known theorem from Kirchhoff uses the Laplacian matrix of $G$ to count the number of spanning trees of G . We discuss a method using vertex contractions to extend this result to spanning forests.

CP6.4 Ramsey Numbers for Connected Colorings of Complete Graphs
Mark Budden (Western Carolina University)
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A well-known result credited to Erdős and Rado states that if a graph is disconnected, then its complement is connected. So, if the edges of a complete graph are colored using two colors, at most one of the subgraphs spanned by edges in each color is disconnected. In 1978, David Sumner considered a variation of Ramsey numbers in which the only 2 -colorings considered are those in which the subgraphs spanned by both colors are connected. This talk will survey some recent results in which connected Ramsey numbers are determined for trees versus trees and will also consider potential ways in which connected Ramsey numbers may be further generalized.

## CP6.5 Trees with minimum weighted Szeged index

Risto Atanasov (Western Carolina University)
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The weighted Szeged index ( $w S z$ ) has gained considerable attention recently because of its unusual mathematical properties. Searching for a tree (or trees) that minimizes the $w S z$ is still going on. In this presentation, we describe several structural details of a tree with minimum $w S z$.

CP6.6 Embedding Grid Graphs on Surfaces - Undergraduate Research Adventures in Topological Graph Theory!
Christian Millichap (Furman University)
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Additional author(s): Fabian Salinas (Furman University and Vanderbilt University)
In an introductory discrete mathematics course, many undergraduate students first encounter the utility problem which translates into asking if $K_{3,3}$ embeds in the plane. This problem can serve as a stepping stone to many fascinating topics in topological graph theory: Kuratowski's Theorem, the Euler characteristic of a surface, forbidden graphs, embedding graphs on surfaces, the genus of a graph, and many more. In this talk, we will describe an undergraduate summer research project that dives deeper into these topological graph theory topics. Specifically, we will analyze embeddings of grid graphs (graph Cartesian products of paths) on closed orientable surfaces and the genera of such graphs. This is joint work with Fabian Salinas (Furman class of 2020, Vanderbilt University).

CP7.1 Gödel's Incompleteness Theorem and the Consistency of Arithmetic: What Every Mathematics Professor Needs to Know
Damon Scott (Francis Marion University)
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The YouTube channel Veritasium is uniformly very good, even excellent, but it is not infallible. A recent episode, "Math's Fatal Flaw," announces that "There is a hole at the bottom of mathematics" and later puts an "X," rather than a check, on the Consistency of Arithmetic. It has caused at least some of its twenty-five million viewers to believe that $1+1$ is not 2 . Every member of the profession should be ready and able to defend our discipline from this well-meaning, but completely erroneous attack. In this talk, we present precisely what Gödel's Incompleteness Theorem is, and what it isn't. Once there is made clear what the line between ground- and meta-mathematics is and how it is important, one easily sees that the famous theorem only shows that there is no proof of the Consistency of Arithmetic within the ground language of Arithmetic, but that meta-proofs of the Consistency of Arithmetic can exist and do.

## CP7.2 What do Elliptic Curves have to do with Congruent Numbers? <br> Melissa Glass (High Point University) <br> mglass@highpoint.edu

Given a square-free positive integer n, does there exist a right triangle with rational sides with area n? This is known as the Congruent Number Problem which dates back thousands of years. We will explore the interesting connections between the Congruent Number Problem, elliptic curves, and a group structure which can be placed on an elliptic curve.

CP7.3 Root Two
Stephen Davis (Davidson College and Charlotte Math Club)
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We survey some delightful approaches to eliminate, again and again, any possibility that $\sqrt{2}$ is rational: laying carpet, balancing masses on a seesaw, folding triangles, reciprocating fractions, and a sequence that approaches zero.

## CP7.4 The Role of Examples in Proof <br> Jordan E. Kirby (Francis Marion University) <br> jordan.kirby@fmarion.edu

During the transition from procedure-based mathematics courses to proof-based mathematics courses, many students struggle to understand the purpose of examples. A productive use of examples can help both the generation and understanding of proofs. Although recent advances in research has argued the importance of productive uses of examples, there is little research investigating the alignment of research with current practices of instructors or of how the instructors view research on example-use. Findings from this study indicate instructors are aligned with mathematics education research on differing levels of example-use. However, many instructors may be hesitant to see examples be used by their students in written work. Despite this hesitation, instructors still encourage students to use examples to aid their understanding of proof.

## CP7.5 My COMPOSTE Experience and the Resulting Activity Sarah Eskew (University of Tennessee Southern) <br> sarklock@utsouthern.edu

The author presents an activity that can be used to introduce the development process of population models to students with minimal perquisite background. Specifically, the students will explore the components of a population model for elk in the eastern US through several small activities to guide them. The activity is a result of participating in the "COnverting Modeling Problems Over to STudent Experiences" (COMPOSTE) workshop in Fall 2023 and is an adaptation of Problem A from HiMCM in 2012. The author will also discuss the process of converting the modeling problem to a classroom activity.

## CP7.6 Experiences with a Data Science Capstone Course

Erin McNelis (Western Carolina University)
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Additional author(s): Kaleigh Benesch, Western Carolina University; John Wagaman, Western Carolina University
In this talk, we introduce the process of how Western Carolina University introduced a Graduate Certificate in Data Science, which has as its culminating experience a course entitled "Data Science Capstone". We will discuss the prerequisites, learning objectives, and deliverables for the course, in addition to course experiences from both a faculty and student perspective. We consider these experiences for the planning of a future undergraduate data science capstone.

## CP8.1 My First Attempt at Mastery-Based Grading <br> Chris Cyr (Covenant College) <br> chris.cyr@covenant.edu

Mastery-based grading is one of many alternative grading practices that have gained traction in higher education over the last decade or two. In this talk, I will discuss how I implemented this grading system for the first time in a Real Analysis course during the Fall 2023 semester, with the goal of providing ideas and advice for anyone interested in applying it in their own courses. I will describe the types of assessments and grading policies I used, evaluate what worked well and what could have gone better, and explain what modifications I made for reattempting this grading system in some of my Spring 2024 classes.

## CP8.2 Grading for Growth in Precalculus <br> Marcela Chiorescu (Georgia College) <br> marcela.chiorescu@gcsu.edu

Standards-based grading (SBG) or grading for growth has the potential to promote a deep understanding of the content while encouraging growth mindset. In this talk, I will discuss my results of a study where I implemented SBG in Precalculus, especially I will focus on how SBG can help students improve their studying habits.

CP8.3 "What's My Grade?" - Some Resources to Help Students Determine Where They Stand in a Standards-Based Graded Course
Rachel Epstein (Georgia College)
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In a course that uses Standards-Based Grading, also known as Mastery Grading, grades are determined by the number of standards or objectives met by the end of the semester. Unlike points-based grading, where grades start at $100 \%$ and can go down and up throughout the semester, in standards-based grading, students build up to a grade and cannot go back down. Because of this, it is hard to tell a student what their grade is midway through the course. In this presentation, I will discuss some resources I use in my Precalculus course to help students understand where they stand, including reflection assignments, a Standards Checklist, and a Grade Possibility Table.

CP8.4 Using Base-5 Number Systems To Teach Future Teachers About Numerical Operations
Kevin LoPresto (Francis Marion University)
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This session will demonstrate how different number systems (base 5 and historical) can be used to teach future early childhood and elementary teachers how young children think about adding, subtracting, multiplying, and dividing numbers. Using Desmos and Polypad, students learn that young children naturally progress through various cognitive levels of problem solving when confronted with novel problems in a number system with which they are not familiar. Historical number systems are used to demonstrate the characteristics of our base 10 and place value-based number system.

CP8.5 Developing Technical Vocabulary: A study in intermediate undergraduate mathematics courses
Lake Ritter (Kennesaw State University)
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Additional author(s): Erik Westlund
As teachers and practitioners, we know that communication is critical for participation in mathematics, and technical vocabulary development is an element of professional preparation. In this talk, we describe a semester long glossary building assignment developed and implemented in select intermediate-level college mathematics courses. We share reflections on the results of student surveys related to the project, the instructional experience, and recommendations for adoption of such a technical vocabulary development intervention.

## CP8.6 Using the quiz-game software Kahoot! in math classes <br> Jennifer Rhinehart (Mars Hill University) <br> jrhinehart@mhu.edu <br> Additional author(s): Laura Steil (Mars Hill University)

This talk will discuss the implementation of Kahoot! software in math courses ranging from elementary geometry (for education majors) to abstract algebra. The software was used to help students gain mastery of definitions. A second objective of using the software was to develop community in the classroom. We will highlight some successes and lessons learned with the implementation.

## 9 Undergraduate Paper Sessions

UT1.1 Cayley Graphs and Their Applications to Group Theory and the BanachTarski Paradox
Kathryn Dover (Presbyterian College)
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A Cayley graph, $C(G, S)$, is a graph formed from a group $G$ and a generating set $S \subseteq G$, where the vertices of the graph are the elements in $G$, and two vertices $a$ and $b$ have an edge between them iff there is a generator $s \in S$ where $a \cdot s=b$. $C(G, S)$ provides a visual representation of many of the group's properties such as associativity and commutativity. In this paper, we explore this relationship between groups and their Cayley graphs. We also look at applications of Cayley graphs including the Banach-Tarski Paradox, which states that a three-dimensional, solid sphere can be decomposed into a finite number of disjoint sets and then reassembled with a group of rotations into two spheres of the same size as the first. The proof for the Banach-Tarski paradox relies on the paradoxical decomposition of the free group $F_{2}$, a decomposition that can be represented visually in the Cayley graph of $F_{2}$.

UT1.2 Almost Positively Curved Eschenburg Spaces Peyton Johnson () peyljohn@ut.utm.edu
We investigate the curvature of Eschenburg spaces with respect to two different metrics, one constructed by Eschenburg and the other by Wilking. We show that, with precisely one exception, that Wilking's metric on the Eschenburg space of cohomogeneity at most two is almost positively curved. This provides the first infinite family of almost positively curved examples since Wilking first introduced his construction. In addition, we completely characterize the curvature of every Eschenburg space with respect to Eschenburg's metric.

UT1.3 Can Normal be Abnormal-Normal?
Gavin Thomas Anderson (Winthrop University)
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Additional author(s): Kylie Zangla: Graduate Student from Winthrop University Dr. Duha Hamed: Professor at Winthrop University
Statistical distributions are used to model data, and can functionally be visualized by thinking of fitting a line on top of a histogram. Many types of distributions have different characteristics, but they all serve the same purpose of modeling data and frequency. For example, the Normal Distribution is a popular distribution, known for its symmetric, unimodal shape and is often used to model symmetric data. Using the Normal Distribution as a base, normal generalization can be defined using the different generalization techniques. The purpose of generalizing distributions is to create a more flexible distribution to model data across a very wide range of applications. Within this paper, a new generalization of a normal distribution is proposed, namely the T-Normal Cauchy, using the T-RYFramework, where T, R, and Y are defined as random variables. For this class of distributions, different mathematical properties were defined. Additionally, three new members of the TNormal Cauchy class of distributions were studied in more detail. One specific member-the Normal-Normal Cauchy distribution-was applied to real-world data to see its competitiveness and flexibility with other established distributions. Our presented class of distributions are found to be strongly capable of fitting unimodal and, at the same time, bimodal data sets.

## UT1.4 Juggling Virtual Braids and Links <br> Ivy Keegan Stump (Davidson College) <br> kestump@davidson.edu

In this talk, we establish a relationship from juggling patterns onto virtual knots: knots that lie in thickened surfaces rather than three-dimensional Euclidean space. To do this, we extend Satyan Devadoss and John Mugno's results on similar mappings in the context of classical links. We accomplish this by defining what constitutes a ladder diagram for juggling patterns with more than two throwing hands. Our formalization has the benefit of realizing virtual knot and link invariants as juggling statistics and giving reduced topological descriptions of juggling and passing sequences.

UT1.5 DNA Self-Assembly Design of Bunk-bed Graphs
Sydney Wilson (Converse University)
cjchang001@converse.edu
Additional author(s): Caleb Chang (undergraduate, Converse University) The flexible-tile model of DNA self-assembly represents $k$-armed branched junction molecules of DNA as vertices with $k$ incident half-edges and desired target nanostructures as discrete graphs. In the context of this model, a central goal is to determine the minimum number of different molecule types that can self-assemble into a given structure represented as a graph. We attempt to determine the minimum number of molecule types for bunk-bed graphs, which are obtained by stacking two copies of a graph and adding edges incident to corresponding vertices. We do so in three different scenarios representing varying levels of laboratory restriction. These three scenarios mandate the use of different techniques and strategies to optimally construct the desired target graphs. We discuss patterns for bunk-bed graphs of cycles as well as bounds, rules, and construction strategies for bunk-bed paths and star graphs.

## UT2.1 Enumerating Visible Tilings of Squares

Cole X Swain (University of Montevallo)
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To tile a square A is to completely fill it with smaller square(s) that have no interior points in common. Such a tiling is said to be visible if each one of the tiles has at least one edge that lies on the perimeter of the square A into which they are embedded. It is relatively simple to show that for any natural number $k \geq 6$, there exists a visible tiling of a square with $k$ smaller squares. We further show that for any even natural number $k \geq 6$, there are at least $2^{\frac{(k-6)}{2}}$ distinct visible tilings of a square.

## UT2.2 3-Dimensional Ferrer's Boards and Their Corresponding Rook and Hit Numbers

Talon Hutto (Brescia University)
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Additional author(s): Kenny Barrese
This talk will discuss the extension from traditional rook placements into three dimensions and what changes this dimensional extension brings when discussing the rook and hit numbers. To do this, we will introduce the general notation for a threedimensional Ferrer's Board and some properties it holds. We will also highlight a significant relation between the rook and hit numbers and explore examples to show this relationship. From this, we can calculate the rook number from the hit number for any board. These topics are wonderful examples of Linear Algebra and Algebraic Combinatorics, which explore the connections between algebraic structures, like polynomials, and discrete structures, like rook placements.

UT2.3 Generalized Delannoy Numbers and Regular Polygons
Jared Bush ()
jaredmartinbush@gmail.com
Additional author(s): Dr. William Griffiths, Faculty overseeing this research This talk will be about our research on Delannoy numbers, and how we look at what Delannoy numbers are and how they are found, and then see where patterns that appear within these numbers show up in other places, particularly by using integer sequences. This talk will be about one specific instance, where one sequence of Generalized Delannoy numbers correlates with a sequence of all possible regions in a convex $(\mathrm{n}+3)$-gon made by non-intersecting diagonals.

## UT2.4 Magic and Antimagic Vertex Labelings of Water Wiggler Graphs Matt Brunet (Winthrop University) <br> brunetm2@winthrop.edu <br> Additional author(s): Juan De Castro Cabrices, Winthrop University, Christian Alter, Winthrop University, Matthew Feldman, University of Michigan, Arran Hamm, Winthrop University, Jessie Hamm, Winthrop University

 A graph with $m$ edges is said to have a magic labeling if its edges can be labeled with distinct numbers from 1 to $m$ so that the sum of the edge labels at each vertex is the same for all vertices. This notion is a generalization of the standard "magic square". Alternatively, a graph with $m$ edges has an antimagic labeling if its edges can be labeled with distinct numbers from 1 to $m$ so that the sums of the edge labels at each vertex are distinct. Antimagic and magic labelings of graphs (and their many generalizations) have been extensively studied.This talk will focus on a family of graphs we call "water wiggler graphs". A water wiggler graph can be obtained by starting with a cycle on $r$ vertices and then duplicating and subdividing edges as much as you like (with every duplicate edge subdivided at least once). We will discuss some results on whether or not graphs in this family have antimagic/magic labelings. If time permits, variations of antimagic/magic-type labelings will be discussed for this family of graphs.

## UT2.5 Star-Critical Ramsey Numbers of Fans Versus Complete Graphs Hayden Privette (Western Carolina University ) whprivette1@catamount.wcu.edu Additional author(s): Mark Budden, Western Carolina University

 Star-critical Ramsey numbers are a refinement for Ramsey numbers. In this talk, we will consider the star-critical Ramsey numbers of fans versus complete graphs. In particular, a general lower bound is determined and multi-color cases are also considered.UT3.1 Time Series Forecasting Models for Local Light Pollution
Abigail Mervine (Winthrop University)
mervinea2@mailbox.winthrop.edu
Additional author(s): Dr. Zachary Abernathy, Winthrop University
This study builds a forecasting model to describe local trends in light pollution: the brightening of the night sky as a result of anthropogenic, artificial light sources. Satellite based radiance measurements can be used as a proxy for light pollution levels. Specifically, a sample from a dataset representing radiance $\left[\left(\mathrm{nW} \cdot \mathrm{cm}^{-2} \cdot \mathrm{sr}^{-1}\right)=\right.$ $\left.10^{-5}\left(W \cdot m^{-2} \cdot s r^{-1}\right)\right]$ values from 2012-2022, collected by the Visible Infrared Imaging Radiometer Suite - Day/Night Band (VIIRS- DNB) sensors on the Suomi NationalPolar Orbiting Partnership (SNPP) satellite, along with time series forecasting, was used to predict future radiance values for Rock Hill, South Carolina. Autocorrelation plots and the augmented Dickey-Fuller test were utilized to select parameters for an Autoregressive Integrated Moving Average (ARIMA) forecasting model. The accuracy of this model, quantified by an Akaike Information Criterion (AIC), was compared to that of models built by Python's auto_arima package, including Seasonal ARIMA (SARIMA) models. The model with the lowest AIC was chosen. A test-train split was then performed on the dataset to cross-validate the chosen SARIMA model. After cross-validation, the chosen model was used to generate an 8 -year (2022-2030) forecast with $95 \%$ confidence intervals, and the forecast appears to show a decrease in radiance values for the Rock Hill area. We conclude with a discussion on the degree to which these radiance values could correlate with changes in light pollution, including the impact of adopting LED technology in artificial lighting over the period of time used in the training data.

UT3.2 Parameter Estimation of a Chromatography Phase in Finite Difference Solutions in MATLAB
Lillian Gensinger (Augusta University)
lgensinger@augusta.edu
Additional author(s): Anastasia Wilson (Augusta University)
We estimate the parameters of finite difference solutions of a chromatography phase in the development of biologics. We do this by comparing the difference of sum of squares with experimental data. This is done in MATLAB using various approaches such as a line-search algorithm, the function fminsearch, and the function fmincon. We developed the approaches using a backward Euler method and later applied the best one to other finite difference solutions for comparison.

## UT3.3 Modeling effects of harvesting-mediated emigration on population persistence Jacob Garrett (Auburn University) <br> jgarret7@aum.edu <br> Additional author(s): Jerome Goddard II, Auburn University Montgomery Trait-mediated behavioral responses (an indirect effect) to other species can affect population dynamics significantly. One example of such a response is modification of emigration probability, which has the potential to change single species persistence, as well as interactions and community structure. Habitat loss and fragmentation due to anthropogenic activities creates landscape-level spatial heterogeneity where remnant patches are often surrounded by a hostile matrix. Matrix composition or hostility is an important component of a landscape and can have profound effects on species movement and boundary behavior, persistence of a single species, and coexistence of interacting species. In this talk, we will introduce a modeling framework to explore effects of harvesting-mediated emigration on population persistence and share some recent results.

## UT3.4 Investigating Tipping Points in the Budyko Climate Model <br> Demmi Ramos (Lenoir-Rhyne University) <br> demmi.ramos@my.lr.edu

The Budyko Climate Model is a dynamical system that represents the change in Earth's temperature. This model considers latitude and assumes that the Earth is only made up of ocean and ice, there are no continents or freshwater, there is no transport/advection, and the Northern and Southern hemispheres are identical. This model has three equilibrium states: stable ice, stable ocean, and bistable. Since dynamical systems can be used to make predictions about future events or behaviors of systems, it is a useful tool when analyzing forecasts of climate change. In this presentation, I will share my work exploring noise-induced tipping points to identify when we might tip from one equilibrium to another. This was done in collaboration with the Mathematics Climate Research Network.

## UT3. 5 Looking for ways to mitigate heat waves in Phoenix, USA <br> Samuel Whitaker (Austin Peay State University) <br> swhitaker $10 @ m y$.apsu.edu <br> Additional author(s): Ramanjit K. Sahi

As a kid, I have always been fascinated to solve problems using mathematics. This passion grew more stronger as I realized the possible applications of math in real world. Later, when I joined the Stem Scholars program at APSU, I was able to see even more applications of mathematics and statistics in real world. In this research, my focus is on studying heat waves in the city of Phoenix, Arizona. In literature, it has been shown that diameter and height of certain trees lead to a cooling effect in urban areas. I am looking at different math/statistical techniques to see which specific trees have more profound effect of mitigating heat waves.

## 10 Undergraduate Poster Session

UP. 1 An Application of Clustering: Addressing Food Insecurity in Western North Carolina
Gracen Alleman (Warren Wilson College)
hrosson@warren-wilson.edu
Additional author(s): Mason Hollar, Warren Wilson College Neylan Visnius, Warren Wilson College
Clustering is a data analysis technique that divides a dataset into distinct groups, with each group containing similar data points. We apply clustering to CDC social vulnerability and health conditions data to identify and map geographic regions in Western North Carolina that are most suitable for targeted outreach efforts by a local food bank.

UP. 2 Mathematical Modeling of COVID-19 Transmission with Focus on Asymptomatic Carriers and Vaccination Schedules
Lauren Beuerle (Elon University)
lbeuerle2@elon.edu
Additional author(s): Dr. Karen Yokley, Elon University
In 2020, the global COVID-19 pandemic erupted. Without knowledge to combat the disease, hospitals around the globe were overrun. Scientists and mathematicians, using past information on extremely infectious viruses, began investigating the effectiveness of social distancing, facial coverings, and eventually, vaccinations. Mathematical models can be used to explore the quantitative effectiveness of vaccinations, facial coverings, and create predictive models to aid the creation of policies in order to prevent future surges in cases. This project utilizes an ordinary differential equation SIR model to explore susceptible (S), infectious (I), recovered $(\mathrm{R})$ populations to explore the different strategies of vaccinations to minimize virus transmission and the impact of asymptomatic individuals. Model output is validated through available CDC data of reported infections during the pandemic. The products of this model can be used as a reference for preventative measures for future epidemics that follow a similar pattern to COVID-19.

## UP. 3 The Probability of Placing a Perfect Bet <br> Evan Carter (Christian Brothers University) <br> ecarter9@cbu.edu

Studying the mathematical foundation behind the probabilities of sports betting in the United States requires exploring various statistical and probability theories. Successful sports betting needs a solid understanding of mathematical principles to make informed decisions and gain an edge in predicting sporting events. In this project we will develop the theory needed to arrive to such informed decisions. The inspiration comes from my own experience with sports betting, in which I have a $69.1 \%$ winning percentage against the odds makers.

## UP. 4 The Circle Problem of Apollonius

Claire DiCugno (King University)
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The classic Circle Problem of Apollonius involves finding a circle, D, that is tangent to three given circles. One way to do so is to see that circle D's center must lie on hyperbolas through any two of the three given circles. We show how to do this numerically and specifically to find circles that are internally tangent to some of the given circles and externally tangent to the others.

## UP. 5 "Temperature" in ChatGPT: An Exploration of Entropy <br> Priscilla Doran (Bryan College) <br> priscillajdoran@gmail.com

This presentation will provide a definition of the temperature parameter in Large Language Models like ChatGPT through the context of Shannon entropy and chaotic dynamics.
Temperature, a feature used to control "creativity" of textual generation in LLMs like ChatGPT, is connected to the inverse temperature factor, a concept borrowed from statistical mechanics, which controls the output of ChatGPT's natural language as anywhere from completely deterministic to completely random.
Using Shannon entropy, we can identify "phase transitions" in ChatGPT's language output, and analyze whether behavior resembles the Markov walk or classical chaos.

## UP. 6 Effects of Multimodal Writing Assignments on Perceptions of Learning in a Calculus I Course <br> Emily Elowitch (Elon University) <br> eelowitch@elon.edu

Additional author(s): Dr. Aaron Trocki, Elon University, Research Mentor This research was designed to assess the effects of multimodal writing on students' perceptions of learning, understanding, and their mathematical ability in Calculus 1. In particular, we investigated to what degree Calculus 1 students perceived multimodal writing as an engaging and effective way to develop their mathematical understanding. We asked Calculus 1 students to complete four writing projects throughout the semester-writing a letter home, creating educational materials, emailing a classmate, and analyzing the accuracy of ChatGPT results-which were strategically designed to increase both their procedural fluency and conceptual understanding of the material. Students completed pre- and post-course surveys, as well as post-project surveys after each assignment about their perceptions of benefits and drawbacks of each project. The results of this research were positive, with students sharing that they felt that the projects deepened and solidified their understanding of the material and that the projects were more enjoyable and engaging than traditional homework assignments. Further, comparing pre- and post-course surveys revealed that students became more confident in their math abilities over the course of the semester.

## UP. 7 Statistical Analysis on the Quality of Life Between Cities in Tennessee Madelynn Fay (Christian Brothers University) <br> mfay@cbu.edu

Statistics is the area of study where data is collected, analyzed and interpreted. Statistical Analysis allows researchers to draw conclusions from the data being interpreted, taking into account trends found in the data, as well as variations. In this project we will compare several quality of life indicators between Memphis, Nashville and Knoxville, prominent cities in Tennessee. Our data includes a diverse range of such indicators. We aim to identify significant differences and similarities between the cities.

UP. 8 Statistical Exploration of Red and White Wine
Katarina Floyd (Francis Marion University)
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Two data sets of red and white Portuguese " Vinho Verde" wine were collected by Paulo Cortez, A. Cerdeira, F. Almeida, T. Matos and J. Reis. The sets include physicochemical and sensory variables. The goal is to statistically analyze and compare the different wines. Areas of interest include acidity levels, pH , and residual sugars.

UP. 9 Picard-Newton: A more robust quadratically convergent nonlinear solver (that just might change your life)
Jessica Franklin (School of Mathematical and Statistical Sciences, Clemson University)
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We consider a 2 -step iteration based on Picard as the first step and Newton as the second, for the purpose of creating a more robust quadratically convergent solver. The new solver enjoys the advantages of both Picard and Newton, as we prove it is quadratically convergent but has a larger convergence basin than Newton. We illustrate the effectiveness of the new method on an analytical test problem as well as the nonlinear Helmholtz equation.

## UP. 10 Modeling Self-Assembly of DNA Nanostructures Resembling Bipartite Graphs Sofia Giraldo (Converse University ) <br> sgiraldojimenez001@converse.edu <br> Additional author(s): Rosalinda Cortez - Converse University and Jordan Rose - Converse University

The bonding properties of DNA make it possible for star-shaped DNA molecules to self-assemble into various intricate nanostructures. These structures can be modeled as discrete graphs, and graph theory techniques can be applied to aid in the discovery of optimal design strategies for desired nanostructures resembling certain graphs. Within the DNA self-assembly graph theoretical framework known as the flexible-tile model, previously known results include a variety of bipartite graphs, including cycle graphs, gear graphs, and many more. To determine whether there are trends among bipartite graphs in this model, we collect and compare alreadyknown results for bipartite graph families and present new results for some additional bipartite graphs.

## UP. 11 The Reality of Math

Sofia Giraldo (Converse University )
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This essay contributes to proving the veracity of mathematics in reality through our daily lives and appearance in daily activities, serving as a unifying force in scientific disciplines that will help corroborate the foundation of our physical existence, ultimately connecting with the philosophical viewpoint of Platonism that underpins realism. This paper investigates the difficulties of viewing mathematics as more than a rigorous discipline taught in schools, highlighting the importance of presenting it as an art form fundamental to reality. The essay delves into the difficulties of students' understanding of mathematical concepts outside the classroom, focusing on the relationship between realism and Platonism about unbodied mathematical theories. The essay presents that mathematical abstract characters can be found within our reality in simple activities and basic projects such as Margaret Wertheim's crocheted coral reefs, which connect complicated mathematical concepts to everyday tasks. This appearance in everyday tasks is examined in this essay, proving that mathematics is fundamentally linked to the experience of reality. Mathematics is the origin of all natural sciences, as every discipline is firmly rooted in math. This essay emphasizes the interaction between mathematics and the natural sciences, particularly physics, with a focus on mathematics' role in substantiating actual theories within the physical universe. The debate present in this paper effortlessly incorporates the Platonist viewpoint, which asserts that several mathematical systems converge under shared axioms, hence substantiating their reality. The example of baking as a practical application of mathematical ideas underlines the importance of mathematics in daily life. Overall, this essay persuasively argues for the actuality of mathematics through its observable manifestations, unifying character across fields, and compatibility with Platonism's philosophical outlook.

## UP. 12 Using Intuition to Teach Mathematical Proofs <br> Kylea Heath (Christian Brothers University) <br> kheath1@cbu.edu

This project describes mathematical proofs for some long-established results, where the approach taken for each one of these proofs is based on visualization of the given mathematical statement, or in principles from physics. Our goal is to investigate if these non-traditional approaches allow for the audience to gain more insight on the statements themselves, and a better understanding of the reasoning in the actual proofs. We will compare traditional proofs with the ones we are presenting in this project, to determine if there are advantages for using proofs where intuition is involved.

## UP. 13 Permanents of Fourier Matrices

James Arthur Hembree Jr. (Lander University )
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Additional author(s): Dr. Chase Worley; Assistant Professor of Mathematics at Lander University
This poster explores computing permanents of discrete Fourier transform matrices. These are Hadamard matrices which have entries given by the discrete Fourier transform. So, this poster will also individually discuss Hadamard matrices and the discrete Fourier transform.

UP. 14 Peer Review Process: AI Generated vs Student Generated Proofs
Abby Hyatt (High Point University )
ahyatt@highpoint.edu
Additional author(s): Jenny Fuselier, High Point University
When first learning how to write mathematical proofs, students often engage in peer review, in part as a way to practice the important skill of critical reading. We used Chat-GPT to create a series of peer-review activities (some containing intentional logical errors) in an introductory proof-writing course. Students compared their experiences critiquing AI-generated proofs with those created by their peers. We present this work and discuss the learning curve associated with prompting ChatGPT in this setting.

## UP. 15 A Bayesian Spectral Analyzer for Biorhythms <br> Emily Jackson (University of Tennessee, Knoxville) <br> ejacks30@vols.utk.edu <br> Additional author(s): Dr. Ioannis Sgouralis, Ph.D

The Discrete Fourier Transform (DFT) is a fast and effective algorithm for conducting Fourier Analysis, but it has rigid input requirements and gives naive results. In this study we introduce a nonlinear regression model which performs Fourier Analysis on periodic and semi-periodic time series. Our model represents the signal underlying the data as a continuous function of time with unknown amplitude and frequency parameters that are fitted to the data. Our statistical approach relaxes the input restrictions of the DFT and provides full posterior distributions for all unknown parameters of interest. We develop a Bayesian model to analyze time series and estimate the parameters of the unknown quantities of our model. Our model is suitable for the analysis of biological signals.

UP. 16 The exploration of adjacency graphs of equivalence classes of planar tanglegrams
Jadon Jones (Berry College)
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Additional author(s): Benji Roush, Ron Taylor
A Tangle is a sculpture turned fidget toy that has a variety of mathematical properties. One natural question that arises from investigating this object is "When is it possible to make a closed version of a tangle lie flat on a table?" These arrangements are called planar tanglegrams. A subsequent question asks for the number of different arrangements of a tanglegram with a given number of pieces and whether these can be obtained from a different configuration under a collection of predefined moves. In this project, we have restricted this collection to two local moves we call rotation and reflection. Our main line of inquiry stems from considering individual tangles as vertices in a graph and saying that two tangles are adjacent if one can be obtained from another by performing a single move. In this poster we present some properties of this adjacency graph, including the bipartiteness and bounds on the maximum degree of a vertex.

## UP. 17 Investigating the Symmetric Subcollection of the Modular Group Cuewon Kim (Vestavia Hills High School) kimcuewon@gmail.com <br> Additional author(s): Jaedeok Kim, Jacksonville State University <br> The modular group $P S L_{2}(\mathbb{Z})=\left\{\left.\left[\begin{array}{l}a \& b \\ c \& d\end{array}\right] \right\rvert\, a, b, c, d \in \mathbb{Z}\right.$ and $\left.a d-b c=1\right\}$ is the col-

 lection of $2 \times 2$ matrices with integer entries and determinant 1 . The two matrices $A$ and $-A$ are considered identical. In this project, we will study the subcollection $S S L_{2}(\mathbb{Z})=\left\{\left.\left[\begin{array}{l}a \& b \\ b \& c\end{array}\right] \in P S L_{2}(\mathbb{Z}) \right\rvert\, a, b, c \in \mathbb{Z}\right.$ and $\left.a c-b^{2}=1\right\}$ of $P S L_{2}(\mathbb{Z})$ consisting of all symmetric matrices. Many interesting number theoretic properties of integers of the form $m^{2}+n^{2}$, where $\operatorname{gcd}(m, n)=1$, can be determined by expressing these symmetric matrices in terms of two generating matrices $R=\left[\begin{array}{l}1 \& 1 \\ 0 \& 1\end{array}\right]$ and $L=\left[\begin{array}{l}1 \& 0 \\ 1 \& 1\end{array}\right]$. A new group can be formed by defining a new binary operation on $S S L_{2}(\mathbb{Z})$. Visualization via the Stern-Brocot tree is an essential part of the study.
## UP. 18 Stopping the Spread: A Novel Computer Virus Model <br> Jackson Lewis (Austin Peay State University) <br> jlewis104@my.apsu.edu

Viruses are typically modeled in a manner of populations, such as susceptible, infected, and recovered, and how they are connected to each other, allowing the spread of the virus. More advanced models include parameters such as limited-duration immunity and infection-induced death. This poster presents a novel model of computer viruses in a network of computers and removable devices when these new parameters, limited-duration immunity and infection-induced death, are introduced. We examine the population equilibria of each group, and how these parameters affect them.

## UP. 19 Transverse-Free Affine Curves

Anna Marti (Georgia College and State University)
anna.marti@bobcats.gcsu.edu
Additional author(s): Shari Hoch (Indiana University of Pennsylvanis; Research Partner at REU), Ethan Soloway (University of Pennsylvania; Research Partner at REU)
In the affine plane over a finite field, an algebraic curve C is said to be transverse-free if every line is either tangent to C or incident with a singularity of C . We present lower and upper bounds on the frequency of transverse-free affine curves in three cases, extending work of Asgarli and Freidin for projective curves. These bounds are distinguished by the number of singularities: we consider smooth curves, curves with one singularity, and curves with any number of singularities.

## UP. 20 Exploring the Mathematics of Sudoku <br> Morgan McCaskill (Francis Marion University) <br> morgan.mccaskill@g.fmarion.edu

Sudoku is all about finding the right numbers to complete the grid. It is a logicbased and combinatorial number-placement puzzle. Here we discuss the history of sudoku, along with the strategy of the game and other sudoku-like puzzles. Finally we introduce some of the mathematics behind the popular puzzle game.

## UP. 21 Key Mathematical Elements of Cryptography <br> Patrick Oweijane (Christian Brothers University) <br> poweijan@cbu.edu

In this project, I will be presenting the mathematical foundation for cryptography which includes important results from number theory. Cryptography is one of the most important tools in modern society, as we rely more and more on technology for every aspect of our lives. The aim of this project is to deepen my understanding of cryptography and its mathematical foundation.

## UP. 22 Predicting Customer Status through Bayesian Networks with Imbalanced Data Handling <br> Bhakti Patel (Francis Marion University) <br> bhakti.patel@g.fmarion.edu

Our project focuses on predicting customer status for a South Carolina company using Probabilistic Graphical Model (PGM) while addressing challenges associated with imbalanced data through different sampling methods. The project's objective is to enhance the accuracy and reliability of predictions, especially in scenarios where imbalances in the dataset could impact the model's performance. We will introduce Bayesian networks, a type of PGM and the implementation of k-fold validation. Explorations into evaluation metrics provide a nuanced understanding of the model's performance. The final stages of the project involve creating graphical representations of key metrics, exploring UnderSampling and OverSampling techniques. The research contributes valuable insights into predicting customer status, particularly in the context of imbalanced data, with the outcomes and methodologies poised for discussion and application in practical settings.

## UP. 23 Polya's Enumeration Formula <br> Nathaniel Pennington (Christian Brothers University) <br> npennin1@cbu.edu

Polya's Enumeration Formula involves counting the number of unique arrangements of the elements of a given set, where symmetry is considered. This project will explore the theoretical foundation of Polya's formula, as well as its applications in different areas of mathematics and other sciences such as chemistry.

UP. 24 Numerical Approximation of Boundary Value Problems with Superlinear Nonlinearity on the Boundary
Larissa Renshaw (University of Tennessee at Martin)
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Additional author(s): Shalmali Bandyopadhyah (UTM) / Thomas L. Lewis (UNC Greensboro)
We approximate solutions numerically for non-linear elliptic boundary value problems with non-linearity on the boundary in one-dimensional setting using the Finite Difference Method. Using MATLAB, we draw bifurcation curves as well as the shapes of solutions.

UP. 25 An Investigation of The Geometric Progression of Rose Curves Naeem Roberts (Francis Marion University)
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It has been well established that the equation for a rose petal and elements associated with the equation can be manipulated to increase the number of petals. For odd scaling factors of theta, the number of petals increase by the scaling factor. For even, the number of petals increases by double the scaling factor. However, literature has not been shown to have investigated the products of sines and cosines and it's rules. Basic treatment of the family of function reveals coordinates mapped in Cartesian that are not only visually stunning but with intriguing behavior when compared to the original rose equation. Treating the rose equation and its parameters as the basis, the family of functions will be studied by investigating the set of points in the cartesian plane that correspond to the set of points corresponding to the polar plane.

UP. 26 ANIMAL MOVEMENT PATTERNS' EFFECT ON DENSITY ESTIMATIONS AND CAMERA TRAP DATA ANALYSIS
Devin Ruppe (Wofford College)
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In order to conserve wildlife and gain valuable insight into wildlife activity, population density estimation is used frequently. There are various ways to estimate population density, two major models being the REM (Random Encounter Model) and REST (Random Encounter with Staying Time) models. However, these estimates are reliant upon camera trapping data, that is subject to possible bias. One way that this data can be inaccurate is under the influence of different animal movement patterns. Three major models of animal movement are Brownian movement, Levy flight, and Ornstein-Uhlenbeck (OU) motion. It is likely that animal movement patterns can change parameters that are used in the REM and REST model, creating significant differences between the two. Here, we modeled a world with camera traps in NetLogo (an agent-based modeling program) in which individuals moved according to one of the three movement patterns mentioned previously. Then, both estimates were implemented into the model to gain simulated estimates and data. Secondly, sensitivity analyses were performed on parameters (i.e. size of world, number of cameras, simulation length, individual movement parameters) to determine, within each type of movement, which estimate was more/less sensitive to changes in said parameters. While this gave us insight into differences within each type of motion, they can also be compared across types of motion to determine whether one type of motion is more/less sensitive to changes in these parameters. Our results indicate that changes in world size, number of cameras, and simulation length do create significant differences within each movement category. However, some of these movements and estimates are less sensitive than others. We can use this information to determine which estimate is best to use in the real world based off of which universal parameters we are least certain about or that could have the highest level of error or variance from test to test.

## UP. 27 An Exploration of Singular Value Decomposition

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Singular Value Decomposition (SVD) is a fundamental concept in linear algebra with widespread applications in various fields such as data analysis, image processing, and signal processing. This project aims to delve into the intricate details of SVD. The exploration will include a comprehensive study and expansion of the concepts and methods on the topic of SVD, as well as some applications.

UP. 28 Predictable Games: How Two Rational Players Settle on Stable Strategies Chris Warner (Christian Brothers University)
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Game theory is the branch of mathematics in which the interaction between two or more participants is modeled mathematically. The objective of game theory is to evaluate strategies and determine which produces the best outcome for each player. In this project, I will examine two-player mathematical games that are modeled by graphs and matrices, demonstrating how strategies are selected under the assumption of rational play.

